Ruimte vwo wiskunde d

de Wageningse Methode

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Theorie

Voorbeeld

Opmerking

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Computer

Echt, moet kunnen

Puzzelen

Pittig

Hint

Facultatief

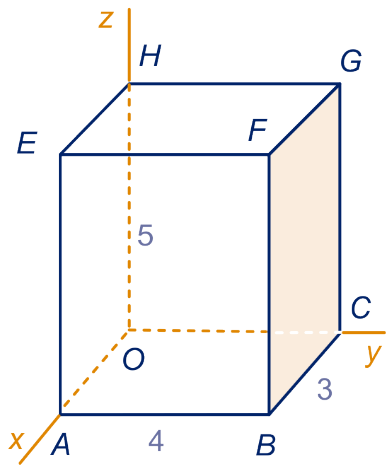

lift
drag

Lijnen in de ruimte

Notaties en afspraken

$A(3,0,0)$ $C(0,4,0)$ $H(0,0,5)$

$\overrightarrow{AB}$ A B



vector $\overrightarrow{AG}$ 3

x 4 y

5 z

$\overrightarrow{AG} = \begin{pmatrix} -3 \\ 4 \\ 5 \end{pmatrix}$ -3 4 5 **kentallen** $\overrightarrow{AG}$

O A $\vec{a}$

$\vec{f} = \begin{pmatrix} 3 \\ 4 \\ 5 \end{pmatrix}$

vlak OEC O E

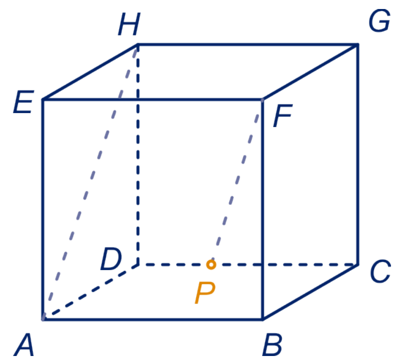
C

E (30,100,50)

1

$ABCD.EFGH$

FP AH



D $A(6,0,0)$ $B(6,6,0)$ $H(0,0,6)$ P

(0,2,0)

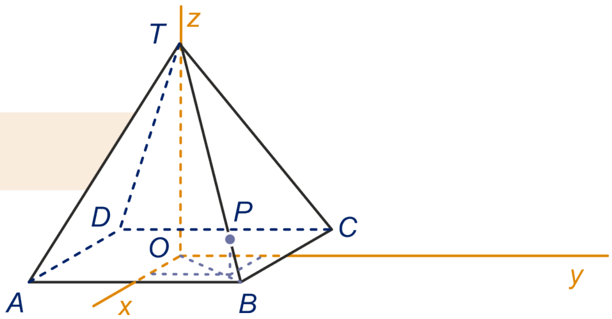
a $\overrightarrow{PF}$ $\overrightarrow{AH}$

b FP AH

2

$T.ABCD$

$A(4,-4,0)$ $B(4,4,0)$ $T(0,0,8)$



a C D

P BT $PT = 3 \cdot BP$

b P

P

c $\overrightarrow{BT} = \begin{pmatrix} \ldots \\ \ldots \\ \ldots \end{pmatrix}$ $TP = 3 \cdot BP$ $\overrightarrow{BP} = \begin{pmatrix} \ldots \\ \ldots \\ \ldots \end{pmatrix}$

B P

x y

z

$P = (4 + \text{-}1, 4 + \text{-}1, 0 + 2) = (3,3,2)$

R BT $BR : BT = 3 : 7$

d R

Opmerking

BT B

BT

BT

$(4 + 4t, 4 + 4t, 0 + 8t)$ t

$(4 - 4t, 4 - 4t, 0 + 8t)$

$(4,4,0) + t \cdot (\text{-}4,\text{-}4,8)$ $(4,4,0) + (\text{-}4t, \text{-}4t, 8t)$

$(x, y, z) = (4,4,0) + (\text{-}4t, \text{-}4t, 8t)$

parametervoorstelling BT

t

BT BT

t

$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 4 \\ 4 \\ 0 \end{pmatrix} + t \cdot \begin{pmatrix} \text{-}4 \\ \text{-}4 \\ 8 \end{pmatrix}$ **vectorvoorstelling** BT

$\begin{pmatrix} \text{-}4 \\ \text{-}4 \\ 8 \end{pmatrix}$ BT

richtingsvector BT

$\begin{pmatrix} \text{-}4 \\ \text{-}4 \\ 8 \end{pmatrix}$ $\begin{pmatrix} \text{-}1 \\ \text{-}1 \\ 2 \end{pmatrix}$ $\begin{pmatrix} 1 \\ 1 \\ \text{-}2 \end{pmatrix}$

t **parameter**

$(a_1 + b_1, a_2 + b_2, a_3 + b_3)$

$(a_1, a_2, a_3) + (b_1, b_2, b_3)$ $(k \cdot a_1, k \cdot a_2, k \cdot a_3)$

$k \cdot (a_1, a_2, a_3)$

Opmerking

De kracht van vectoren

3

a BT y z

$(x, y, z) = (0,0,8) + t \cdot (-1,-1,2)$ $(x, y, z) = (-t, -t, 8 - 2t)$

BT

b

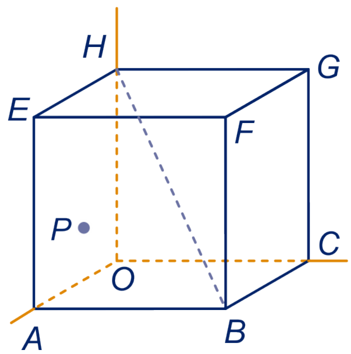
c AT

4

$ABCO.EFGH$ $A\,(4,0,0)$ $C\,(0,4,0)$

$H\,(0,0,4)$ P

BHP V



a V

AE

k P BH

b k

P $(4,1,2)$

c

k

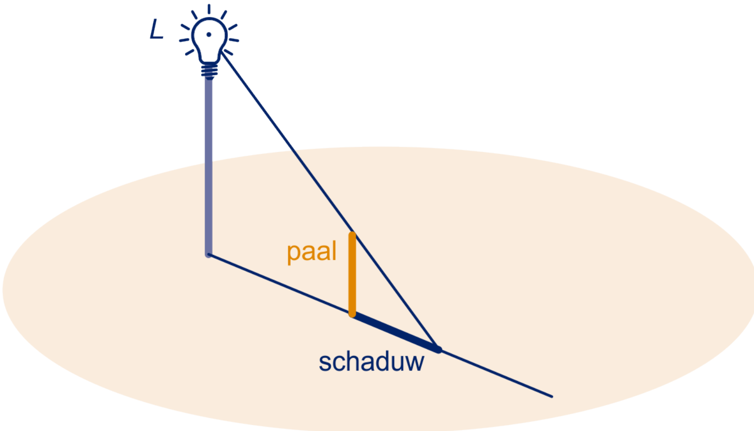
m P AC

d m

e

Opmerking

L

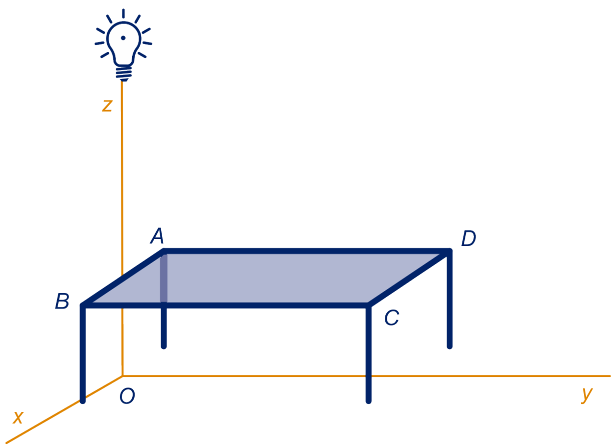


5

80 120 40

A B C D

AB 120



a

b

$L\,(0,0,12)$

$A = (\text{-}4,0,4)$ $C = (4,12,4)$

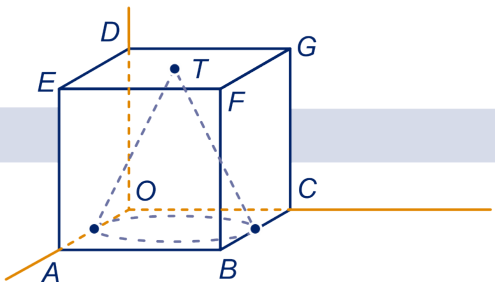
c LC
C

$(2,3,4)$

d

6

$OABC.DEFG$ $A(40,0,0)$ $B(0,30,0)$
$D(0,0,40)$ $T(20,15,40)$
Oxy 15
OF S U
O S



a S U

b $OBFD$
S U

c S U

S U

OB P Q P
O

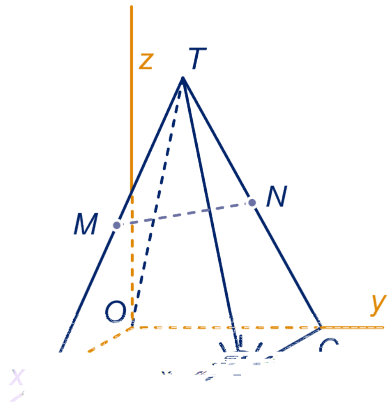
d P

e TP

OF

f
S

7
$T.OABC$ $A(4,0,0)$
$C(0,4,0)$ $T(2,2,6)$ M AT N
CT B



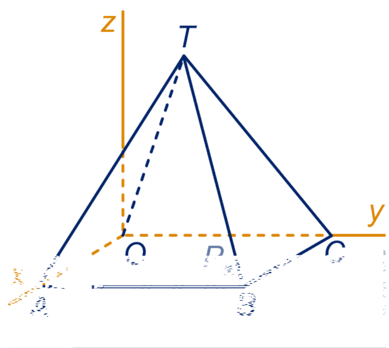
a MN OAT
OCT

b OT
t
B MN
s

c s t

d

8
$T.ABCO$ $A(6,0,0)$
$C(0,6,0)$ $T(3,3,6)$
$P(4,5,0)$



a TAP

b P TA

S TBC z

c S

d S

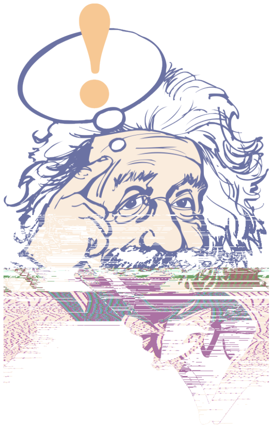
9

(4,12,0) (0,-7,1) (8,2,0)
(-4,1,1)

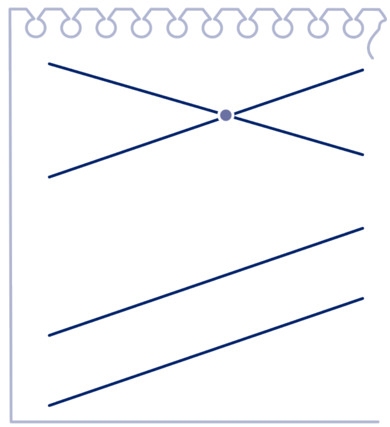
a
b

c

Opmerking

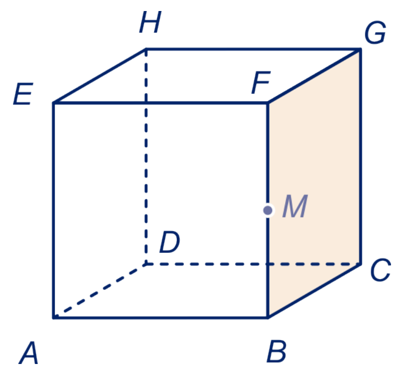


Onderlinge ligging van lijnen



90°

10 $ABCD.EFGH$ M
BF



BG AH
BG ED
HM BD
AM GH

BG ED

Definitie

hoek van twee (kruisende) lijnen

Voorbeeld

BG AH AH

ED BG ED

11

a DF

b AC GM

AB HM

AF DE

Het inproduct

12

$ABCO.EFGH$

A

x C y H z

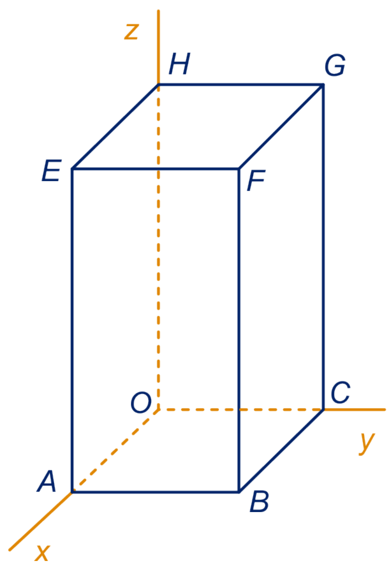
a OF 3

4 5 x y z

b OF p

q r

c $\overrightarrow{BH}$



$X = (x, y, z)$ $\overrightarrow{OX} = \sqrt{x^2 + y^2 + z^2}$

x y z

$\vec{v}$ $\left|\vec{v}\right|$

13

$P(1,2,3)$ $Q(-1,3,-3)$

a $\overrightarrow{PQ}$ $\left|\overrightarrow{PQ}\right|$

$A = (a_1, a_2, a_3)$ $B = (b_1, b_2, b_3)$

b $\overrightarrow{AB}$ $\left|\overrightarrow{AB}\right|$

$A = (a_1, a_2, a_3)$ $B = (b_1, b_2, b_3)$ $\overrightarrow{AB} = \begin{pmatrix} b_1 - a_1 \\ b_2 - a_2 \\ b_3 - a_3 \end{pmatrix}$

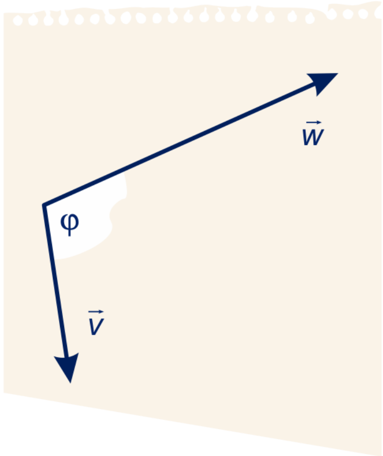
$\left|\overrightarrow{AB}\right| = \sqrt{(b_1 - a_1)^2 + (b_2 - a_2)^2 + (b_3 - a_3)^2}$

inproduct $\vec{v}=\begin{pmatrix}v_1\\v_2\\v_3\end{pmatrix}$ $\vec{w}=\begin{pmatrix}w_1\\w_2\\w_3\end{pmatrix}$ $v_1\cdot w_1+v_2\cdot w_2+v_3\cdot w_3$

$\vec{v}$ $\vec{w}$ $\vec{v}\cdot\vec{w}$

$\vec{v}\cdot\vec{w}=\left|\vec{v}\right|\cdot\left|\vec{w}\right|\cdot\cos(\ )$

$\vec{v}$ $\vec{w}$



Voorbeeld

$(x,y,z)=(1+t,2-t,3+2t)$

$(x,y,z)=(1-2t,2+4t,3)$

$\vec{v}=\begin{pmatrix}1\\-1\\2\end{pmatrix}$ $\vec{w}=\begin{pmatrix}-2\\4\\0\end{pmatrix}$

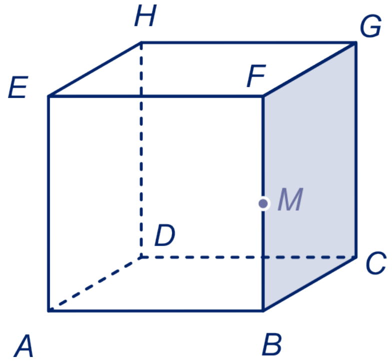
$\vec{v}\cdot\vec{w}=\left|\vec{v}\right|\cdot\left|\vec{w}\right|\cdot\cos(\ )$ $\vec{v}\cdot\vec{w}=-6$ $\left|\vec{v}\right|\cdot\left|\vec{w}\right|=2\sqrt{30}$

$\cos(\ )=\frac{-6}{2\sqrt{30}}=-0{,}547....$ $=123°$

$57°$

14

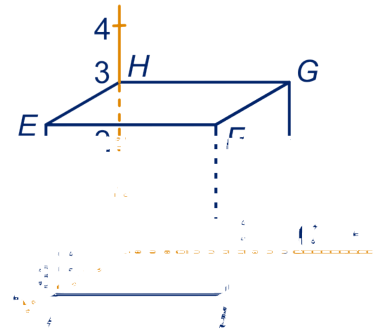
AC GM AB HM AF DE



15 $ABCO.EFGH$ 3

a AC

OF



P FB P $(3,3,z)$

z 0 3

b z OP EC

c z AG HP

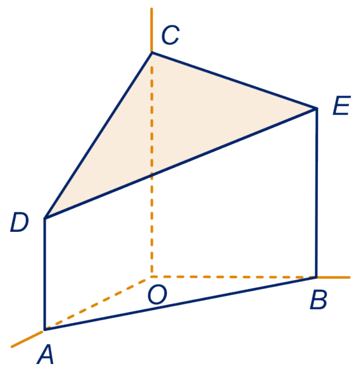
d

OF

16

OAB

CDE



$O = (0,0,0)$ $A = (6,0,0)$ $B = (0,6,0)$ $C = (0,0,8)$ $D = (6,0,4)$ $E = (0,6,6)$

a

DE X DE CX

b X

Parametervoorstelling van een vlak

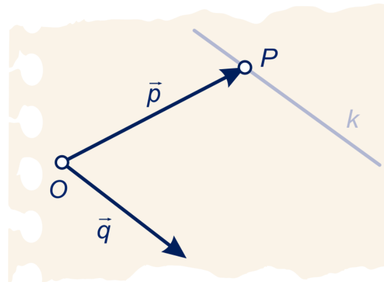
k $\vec{q}$ k

P k k P

$\vec{q}$

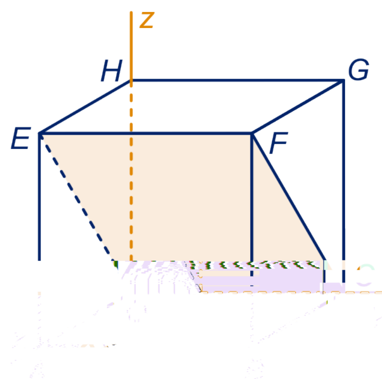
$\vec{x}=\vec{p}+t\cdot\vec{q}$ t

k O



k k t

17



2 V O C E

V

$\vec{r}=\begin{pmatrix}1\\0\\1\end{pmatrix}$ $\vec{s}=\begin{pmatrix}0\\1\\0\end{pmatrix}$

V O

$p\cdot\vec{r}+q\cdot\vec{s}$ p q

a p q

b p q

EF

(7,-10,7) V

c p q

$\vec{x}=p\cdot\vec{r}+q\cdot\vec{s}$ p q

V

V $(x, y, z) = (p, q, p)$

18

V $\vec{k}$

W AB W

W $\vec{x}=\vec{k}+p\cdot\vec{r}+q\cdot\vec{s}$

a W

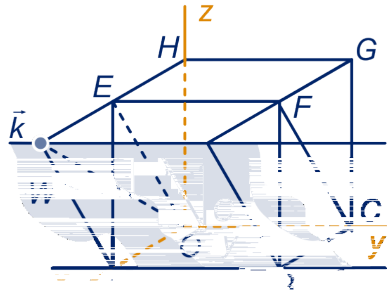
m $(x,y,z)=(t,2-2t,2t)$

b m

m W

c

W m



19

U $$\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}2\\0\\2\end{pmatrix}+p\cdot\begin{pmatrix}-1\\1\\0\end{pmatrix}+q\cdot\begin{pmatrix}0\\1\\-1\end{pmatrix}$$

a U

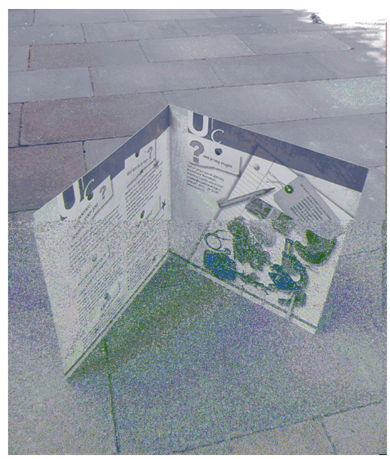
b U

OF U S

c S

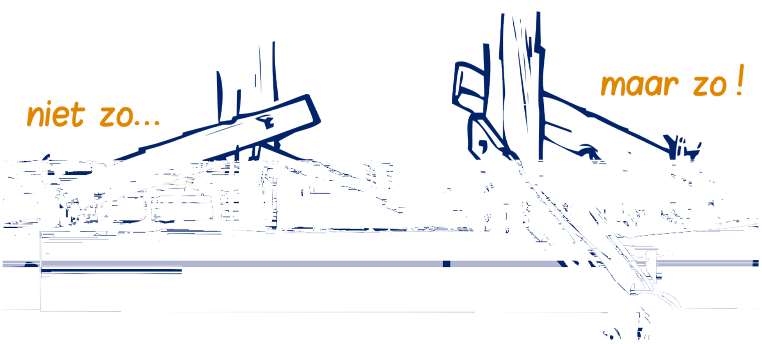
d S

Vergelijkingen van vlakken

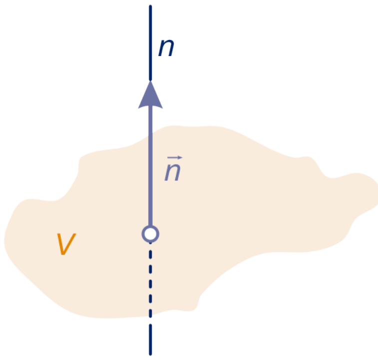


twee

elke



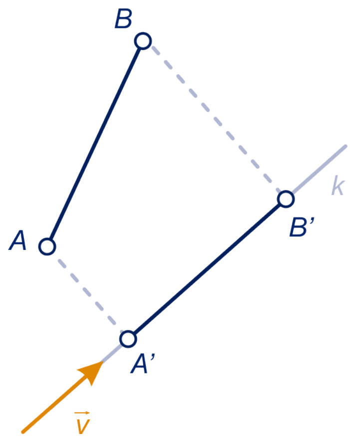
n V V
90°
V
n **normaal** V $\vec{n}$ n
normaalvector V



Stelling

AB k

$$\frac{\left|\overrightarrow{AB}\cdot\vec{v}\right|}{\left|\vec{v}\right|}$$



k
$\overrightarrow{AB}$
k

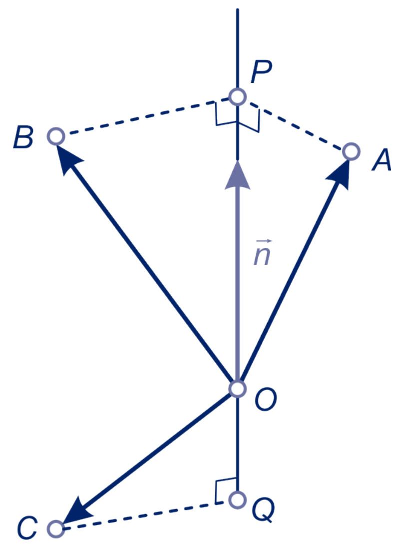
20

n O $\vec{n}$
n 4 A B
P n OP 6
C n Q OQ 2
$\vec{a}$ $\vec{b}$ $\vec{n}$ $\vec{c}$ $\vec{n}$

a $\vec{n}\cdot\vec{a}$ $\vec{n}\cdot\vec{b}$ $\vec{n}\cdot\vec{c}$

X $\vec{x}$ $\vec{n}$ $\vec{a}$
P n

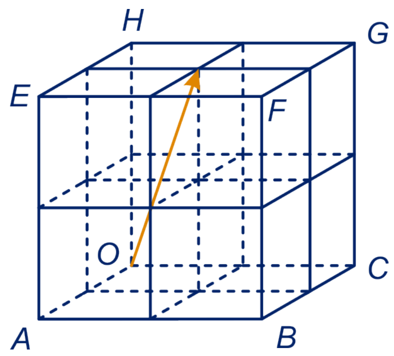
b X



Stelling

n $\vec{n}$ A
X $\vec{n}\cdot\vec{x} = \vec{n}\cdot\vec{a}$
V A n n $\vec{n}$
V

21 $ABCO.EFGH$ 2 $A(2,0,0)$
$C(0,2,0)$ $H(0,0,2)$ N



$X(x, y, z)$ $\vec{n} \cdot \vec{x} = \vec{n} \cdot \vec{a}$
V

a X $V \Leftrightarrow x + y + 2z = 2$

b (x, y, z)
$x + y + 2z = 2$
V

$y + 2z = 2$
B
$\vec{n} \cdot \vec{x} = \vec{n} \cdot \vec{b}$ $\vec{n}$

c $\vec{n}$

$y + 2z = 2$
W B $\begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}$

d W
W

e $y + 2z = 2$
x

f $(x, y, z) = (\ldots, \ldots, \ldots)$
V W $(x, y, z) =$
$(\ldots, \ldots, \ldots)$ V W

(x, y, z) $ax + by + cz = d$ $\vec{n} = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$
$\vec{n}$

$a = 0$ $b = 0$ $c = 0$
x y z

Opmerking

Een vlak en zijn snijpunten met de coördinaat-assen

22

$ABCO.EFGH$ 4 V

$x + y + z = 6$

a V

b V

$$\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

V $\overrightarrow{OF}$

OF

V

c OF

BG

BG $OCFE$ OF

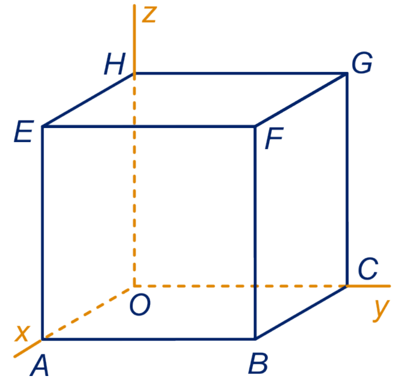
BG

EG N

OF V OF

BN

d $OBFH$



23

U

$x + y + 2z = 6$

a U

b U

c U

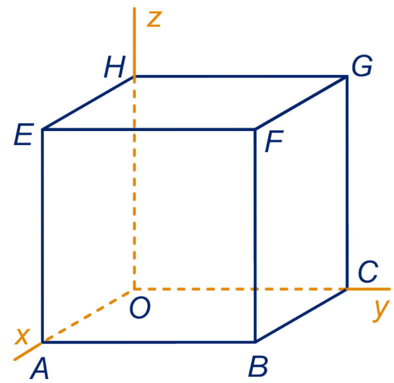
M BF N EG

$\overrightarrow{ON}$ U

ON U

d ON EG

e $OBFH$ ON MN



Opmerking

A B C

ABC

P V

PQ

PQ V Q V

24

U MEG

a

b O EGM

O EGM

O ACH

$OBFH$

c

d O ACH

25

$\frac{x}{3} + \frac{y}{4} + \frac{z}{5} = 1$

V

a A C H V

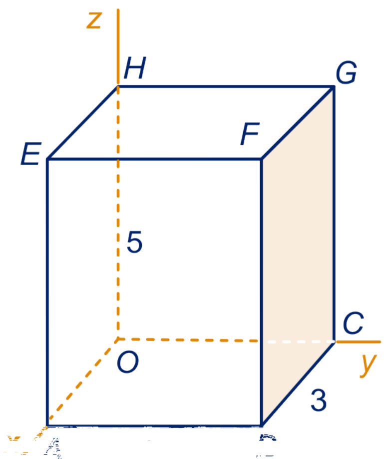
$ax + by + cz = d$

V

b

W $\frac{x}{3} + \frac{z}{5} = 1$

c



W y

W

d

W y

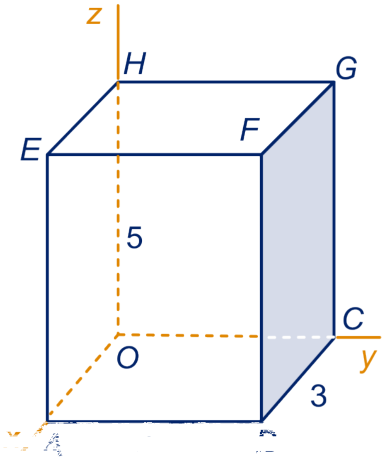
$(3,0,0)$ $(0,0,5)$ ABH

e ABH

a b c 0

- $(a,0,0)$ $(0,b,0)$ $(0,0,c)$ $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$
- x y $(0,b,0)$ z $(0,0,c)$ $\frac{y}{b}+\frac{z}{c}=1$
- Oxy z $(0,0,c)$ $\frac{z}{c}=1$

26



a BEG

b BEG BEG

c BEH BEH

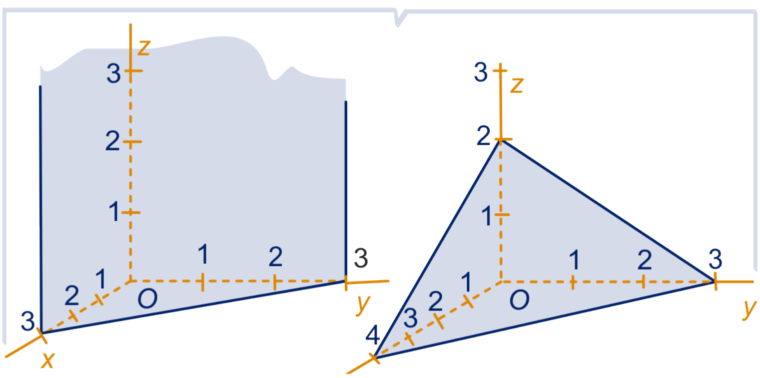
d O BEH

Opmerking

$x+y=3$

z

$3x+4y+6z=12$



27

$2x + 3y + 12z = 6$ $2x + 3y = 6$ $\frac{x}{2} + \frac{y}{4} + \frac{z}{3} = 1$

$x + 3z = 3$ $x - y = 0$ $2x = 4$

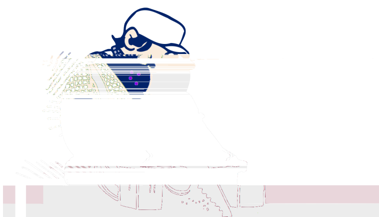
28 $ABCO.EFGH$ 3 A C H

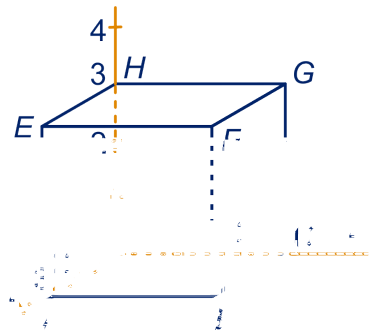
V $2x + y + z = 4$

a V

V

V HG





b

c V

V

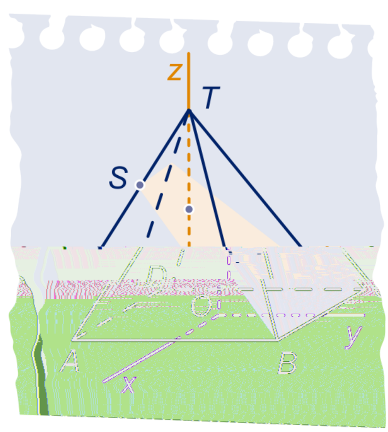
d

29 $T.ABCD$

$A(3,\text{-}3,0)$ $B(3,3,0)$ $C(\text{-}3,3,0)$ $D(\text{-}3,\text{-}3,0)$ $T(0,0,6)$ V

$y + z = 3$

V AT



30 U $2x + 3y + 4z = 6$ V

$4x + 6y + 8z = 17$

a V

$2x + 3y + 4z = \ldots$

b U V

U V

W $x + ay + bz = 20$

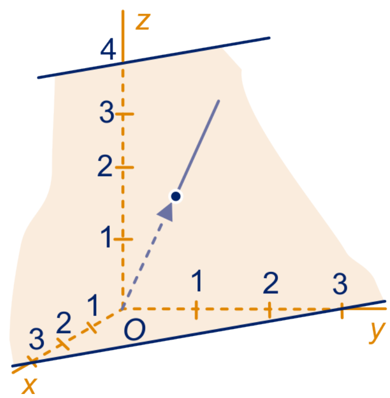
a b

U W

c a b

31 **Kogel door de tent**

$(3,0,0)$ $(0,3,0)$ $(0,0,4)$



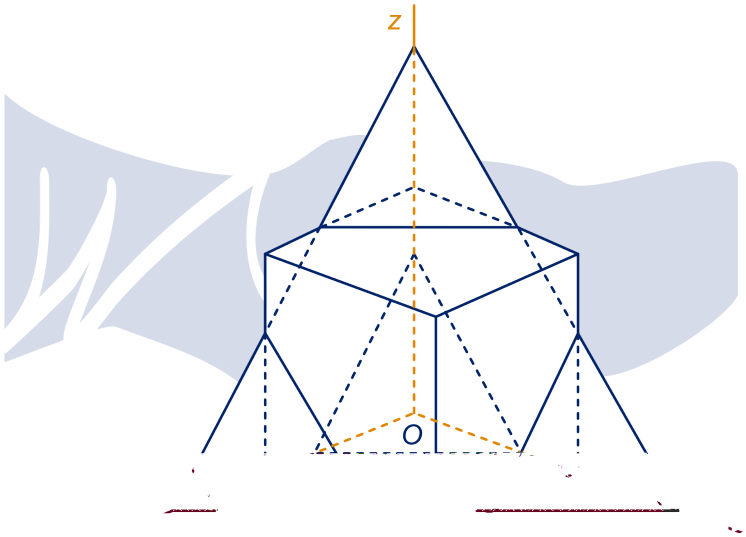
a

$O(0,0,0)$

$$\begin{pmatrix}1\\1\\2\end{pmatrix}$$

b

32



6

a

b (6,0,0)

Vergelijking van een vlak met behulp van een pv

Voorbeeld

V $A(2,3,1)$ $B(4,3,2)$ $C(0,0,3)$

$\vec{n}$ V

V $\overrightarrow{AB} = \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}$ $\overrightarrow{AC} = \begin{pmatrix} -2 \\ -3 \\ 2 \end{pmatrix}$

b $\vec{n} = \begin{pmatrix} 1 \\ b \\ -2 \end{pmatrix}$ $\overrightarrow{AB}$

b $\vec{n}$ $\overrightarrow{AC}$

$\vec{n} \cdot \overrightarrow{AC} = 0 \Leftrightarrow -2 \cdot 1 + -3 \cdot b + 2 \cdot -2 = 0$

$\vec{n} = \begin{pmatrix} 1 \\ -2 \\ -2 \end{pmatrix}$ V

V $x - 2y - 2z = d$

d

V A

$x - 2y - 2z = -6$ V

33

- (1,2,3) (1,4,5) (2,6,0)
- (1,2,3) (2,4,5) (2,4,0)
- (1,2,3) (2,4,5) (-4,1,3)
- (1,2,3) (4,2,3) (2,6,0)

0

Voorbeeld

V $A\,(1,2,3)$ $B\,(3,1,4)$ $C\,(0,4,5)$

V $\overrightarrow{AB} = \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$ $\overrightarrow{AC} = \begin{pmatrix} -1 \\ 2 \\ 2 \end{pmatrix}$

V

$\overrightarrow{AB} + 2 \cdot \overrightarrow{AC} = \begin{pmatrix} 0 \\ 3 \\ 5 \end{pmatrix}$ a $\vec{n} = \begin{pmatrix} a \\ 5 \\ -3 \end{pmatrix}$

$\begin{pmatrix} 0 \\ 3 \\ 5 \end{pmatrix}$ a $\vec{n}$

$\overrightarrow{AB}$ $\overrightarrow{AC}$

$\overrightarrow{AB} \cdot \vec{n} = 0 \Leftrightarrow a = 4$ V

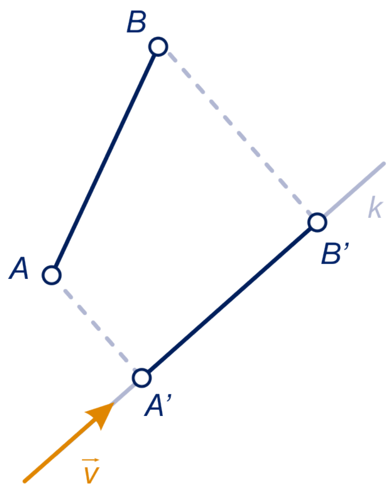
$4x + 5y - 3z = d$ (1,2,3) V $d = 5$

V $4x + 5y - 3z = 5$

Opmerking

34

- (1,2,3) (2,1,-3) (7,0,5)
- (3,0,0) (0,-2,1) (5,2,2)
- (1,2,3) (1,0,0) (4,1,1)
- (1,1,3) (3,5,5) (0,-1,2)



AB k

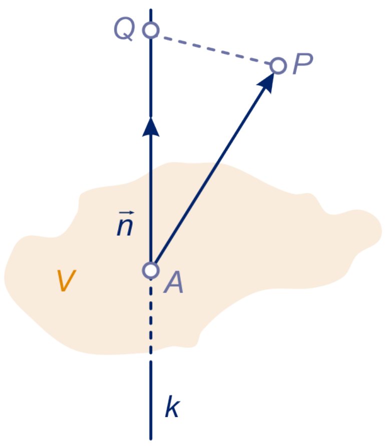
$\vec{v}$ k

$$\frac{\left|\overrightarrow{AB}\cdot\vec{v}\right|}{\left|\vec{v}\right|}$$

35

V $2x-3y+4z-11=0$ P

(1,2,3) $\vec{n}=\begin{pmatrix}2\\-3\\4\end{pmatrix}$ V



$A\,(1,-3,0)$ V

a $\vec{n}\cdot\overrightarrow{AP}$ $\left|\vec{n}\right|$

$\overrightarrow{AP}$ k A $\vec{n}$

P V

b

$\vec{n}\cdot\overrightarrow{AP}=\vec{n}\cdot\vec{p}-\vec{n}\cdot\vec{a}$ $\vec{n}\cdot\overrightarrow{AP}$

P

$2x-3y+4z-11$

c

P V

P $2x-3y+4z-11$

$$\left|\begin{pmatrix}2\\-3\\4\end{pmatrix}\right|$$

V $\vec{n}$ P V

P V

A V k A
$\vec{n}$ P V AQ
Q P k

$$\frac{\left|\overrightarrow{AP}\cdot\vec{n}\right|}{\left|\vec{n}\right|}$$

$A=(a_1,a_2,a_3)$ $P=(p_1,p_2,p_3)$ $\vec{n}=\begin{pmatrix}n_1\\n_2\\n_3\end{pmatrix}$

V $n_1x+n_2y+n_3z-d=0$

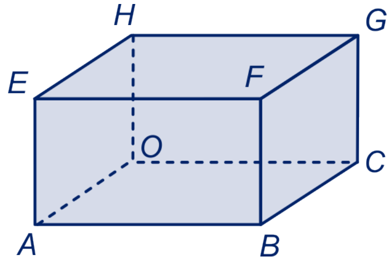
d

$\vec{n}\cdot\overrightarrow{AP}=n_1\cdot(p_1-a_1)+n_2\cdot(p_2-a_2)+n_3\cdot(p_3-a_3)$
$=n_1\cdot p_1+n_2\cdot p_2+n_3\cdot p_3-(n_1\cdot a_1+n_2\cdot a_2+n_3\cdot a_3)$
$n_1\cdot p_1+n_2\cdot p_2+n_3\cdot p_3-d$ $n_1\cdot a_1+n_2\cdot a_2+n_3\cdot a_3=d$
A V

V $n_1\cdot x+n_2\cdot y+n_3\cdot z-d=0$
P (p_1,p_2,p_3) P V
$$\frac{\left|n_1\cdot p_1+n_2\cdot p_2+n_3\cdot p_3-d\right|}{\sqrt{{n_1}^2+{n_2}^2+{n_3}^2}}$$

Voorbeeld

$A(7,0,0)$ $C(0,7,0)$
$H(0,0,4)$ F ACH
ACH $\frac{x}{7}+\frac{y}{7}+\frac{z}{4}=1$
$4x+4y+7z-28=0$
$F(7,7,4)$ $\frac{|4\cdot 7+4\cdot 7+7\cdot 4-28|}{\sqrt{4^2+4^2+7^2}}=6\frac{2}{9}$



36

$O(0,0,0)$ $x+2y+3z=10$
$A(1,2,3)$ $x+2y-3z=12$
$A(1,2,3)$ $(2,2,\text{-}3)$ $(0,6,0)$
$(6,3,0)$

37

k $(1,\text{-}1,2)$ $(3,3,3)$ V
$x+2y-2z=12$
a k
b k 3 V

38

$x + y + z = 3$

$x + y + z = 9$

39

4

2 3

a ACH

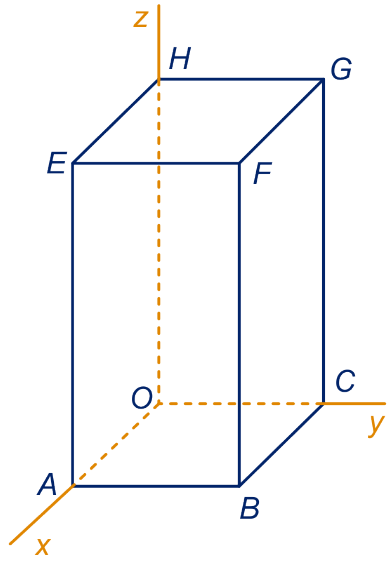
b O ACH

$OACH$ $\frac{1}{3} \cdot \frac{1}{2} \cdot 4 \cdot 3 \cdot 2 = 4$

c

ACH

d ACH



40

$ACHF$

ACH

F

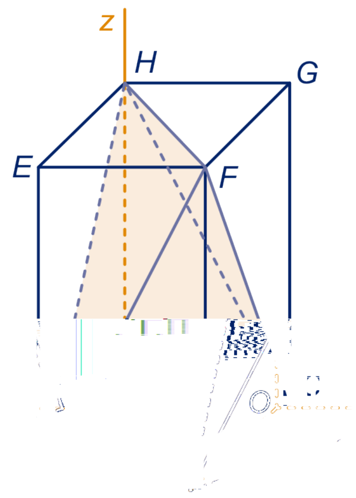
ACH ACH

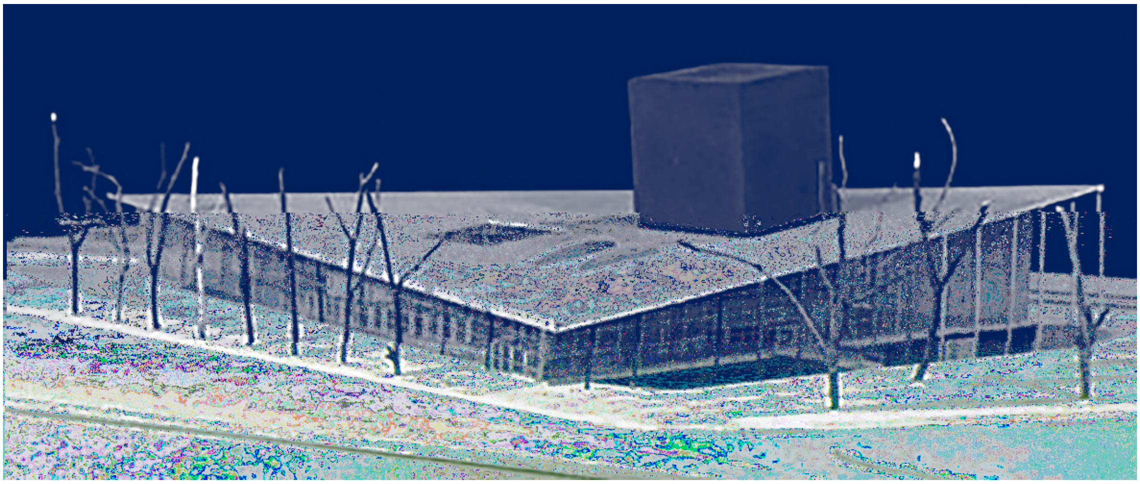
a F ACH

b $ACHF$

$ACHF$

c





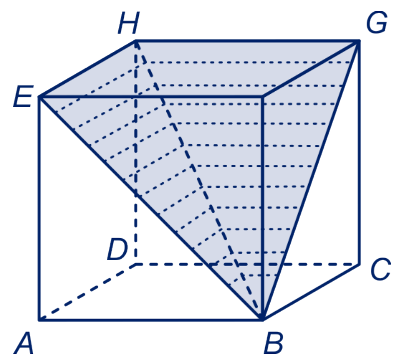
De hoek van twee vlakken

41

45°

a 45°

b 90°

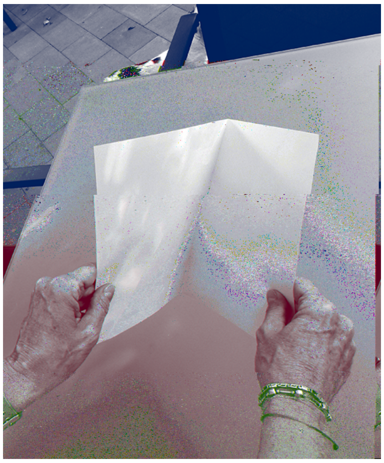


ABCD.EFGH *BEH*
BGH

c

42

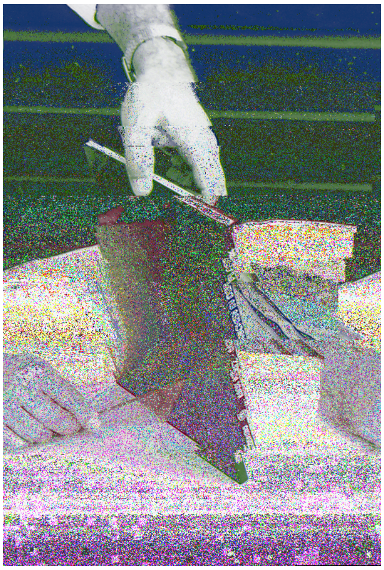
a



b

43

a

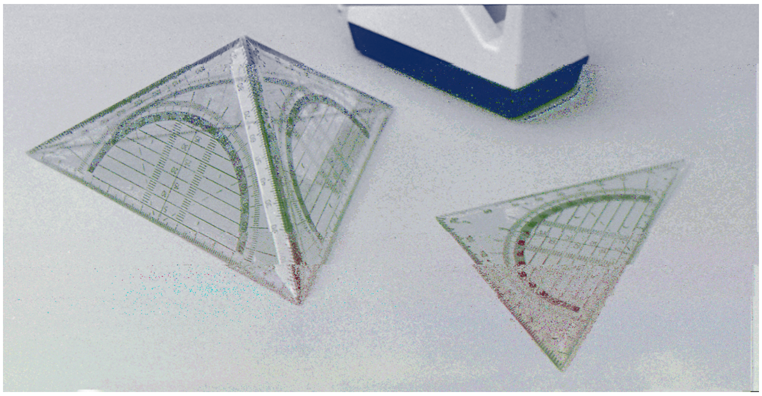


b

45˚

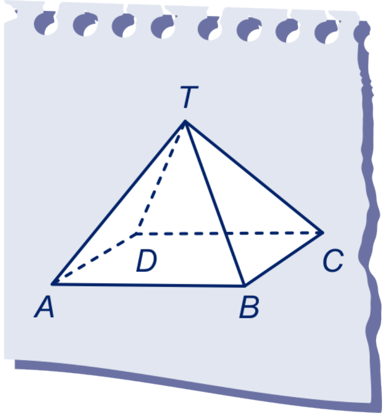
45˚ 90˚

44



45

ABCD.T 6
ADT

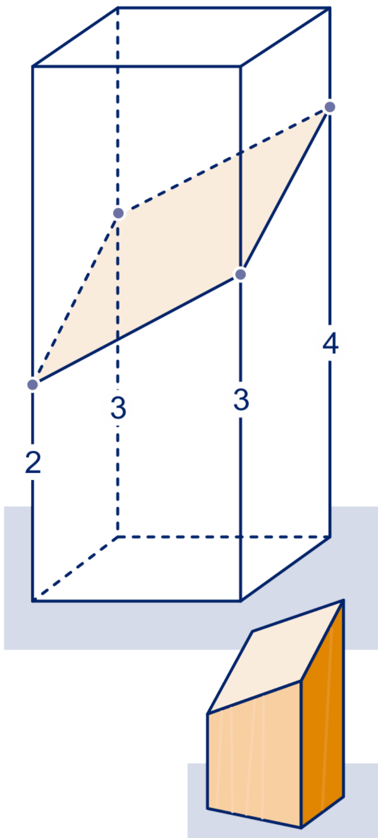


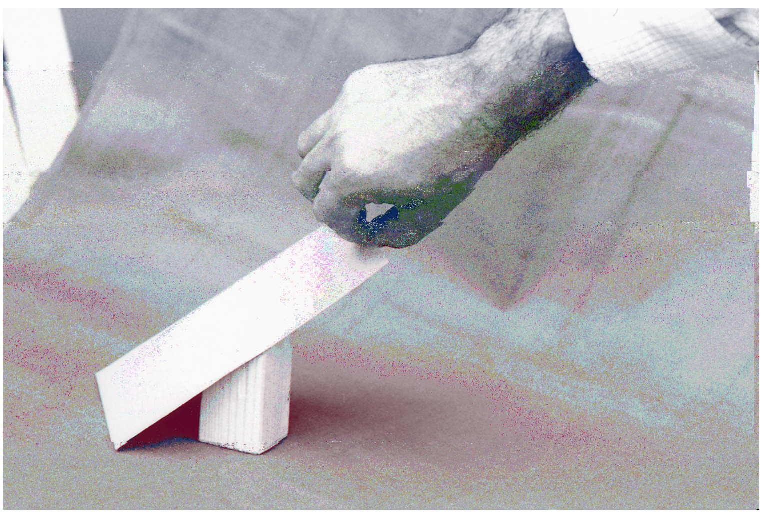
46

2 2

2 3 4 3

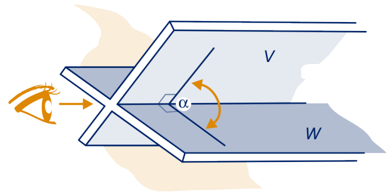
a
b





V W

standhoek V W



47

V W
45°

a

- V W
  77°
- V W
  45°
- V W
  27°

l V m W

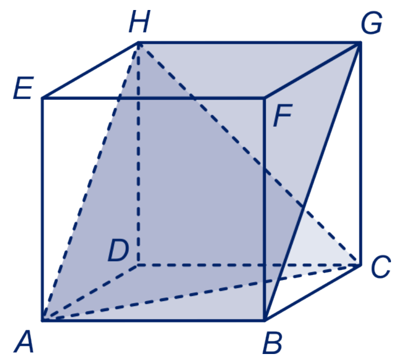
b l m

48

$ABCD.EFGH$
ACH $ABGH$ AH

a

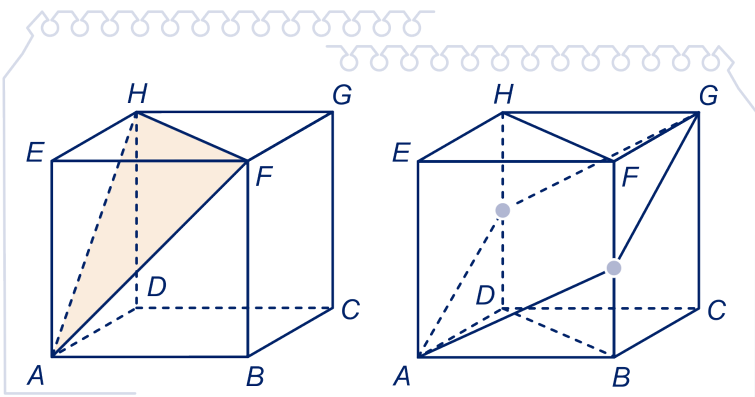
b



49

$ABCD.EFGH$

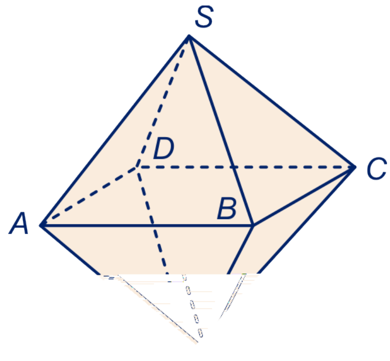
a AFH



N HD M FB

b $AMGN$ $BFHD$

50 $ABCD.ST$

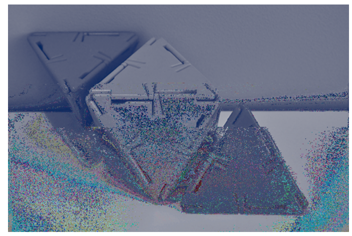


a
TAD TBC

b
TAD TCD

51

a

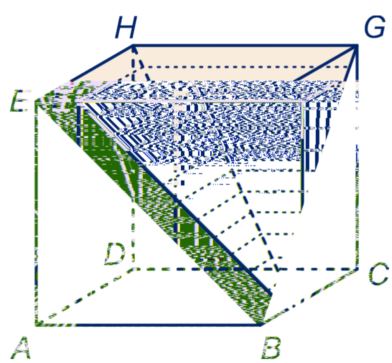


b

Opmerking

$109°$

52 **De vouw in de kilgoot**



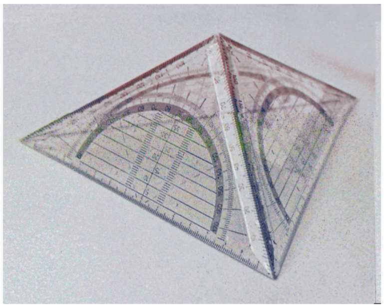
BEH BGH $ABCD.EFGH$
P BH EP BH
6

a BP
$BP : HP$

b EPG

53

De drie geodriehoeken



16

90˚

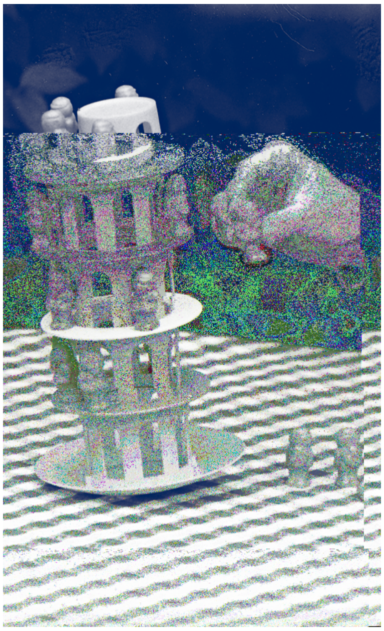
a

b

c

De hoek tussen een lijn en een vlak.

54



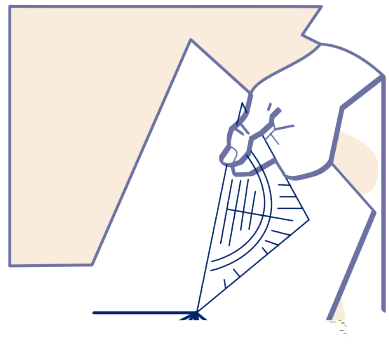
a

b

4 54

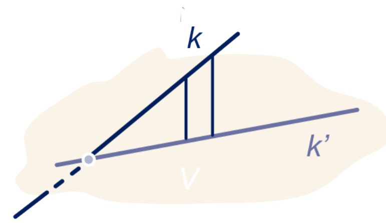
c

55



- 77°
- 45°
- 23°

k V k V
k' k V
k k' k
V



56

$ABCD.EFGH$
A B G V
EC
V
E V N

a N

EC V
EG V M

b M

57

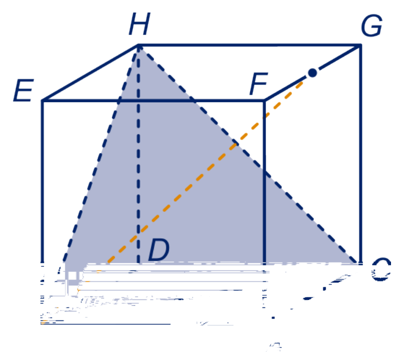
$ABCD.EFGH$ 6 M
FG AM
V A C H
$D = (0,0,0)$
$A = (6,0,0)$ $C = (0,6,0)$ $H = (0,0,6)$

a V

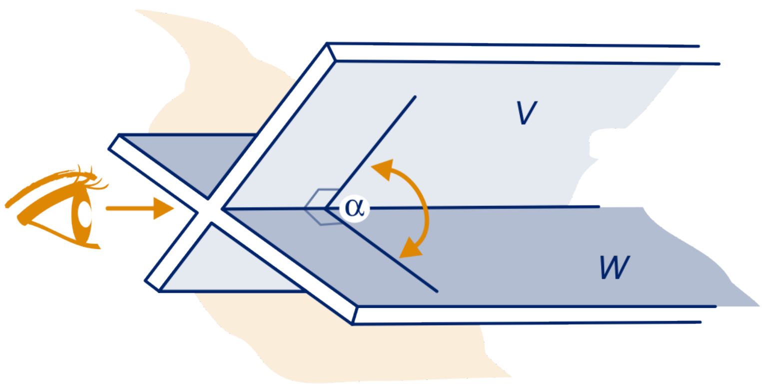
M V N

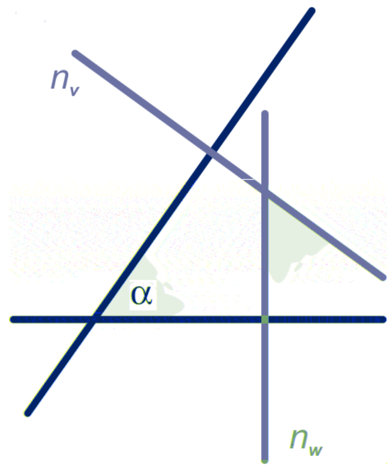
b N

c AM V



Hoeken berekenen met normalen





V W

n_V n_W n_V n_W V W

Opmerking

•

V $(6,0,0)$ $(0,2,0)$

$(0,0,5)$

$\frac{x}{6} + \frac{y}{2} + \frac{z}{5} = 1$ $5x + 15y + 6z = 30$

V $\begin{pmatrix} 5 \\ 15 \\ 6 \end{pmatrix}$ V

W x $(6,0,0)$ z $(0,0,2)$ y

$\frac{x}{6} + \frac{z}{2} = 1$ $x + 3z = 6$ W

$\begin{pmatrix} 3 \\ 0 \\ 1 \end{pmatrix}$ W

-

U

$$\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}3\\1\\2\end{pmatrix}+s\cdot\begin{pmatrix}3\\3\\2\end{pmatrix}+t\cdot\begin{pmatrix}3\\4\\-1\end{pmatrix}$$

U $\vec{s}=\begin{pmatrix}3\\3\\2\end{pmatrix}$ $\vec{t}=\begin{pmatrix}3\\4\\-1\end{pmatrix}$

U

0 $\vec{u}=\vec{s}+2\vec{t}=\begin{pmatrix}9\\11\\0\end{pmatrix}$

$\vec{n}=\begin{pmatrix}11\\-9\\a\end{pmatrix}$ $\vec{u}$ a

$\vec{n}\cdot\vec{u}=0$

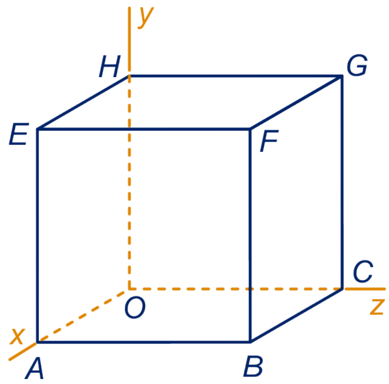
a $\vec{n}\cdot\vec{s}=0$ $a=-3$

$\vec{n}=\begin{pmatrix}11\\-9\\-3\end{pmatrix}$ U

U $\vec{u}$ $\vec{s}$

Voorbeeld

$ABCO.EFGH$ $A\,(1{,}0{,}0)$ $C\,(0{,}1{,}0)$
$H\,(0{,}0{,}1)$ $BCHE$ $ABHG$

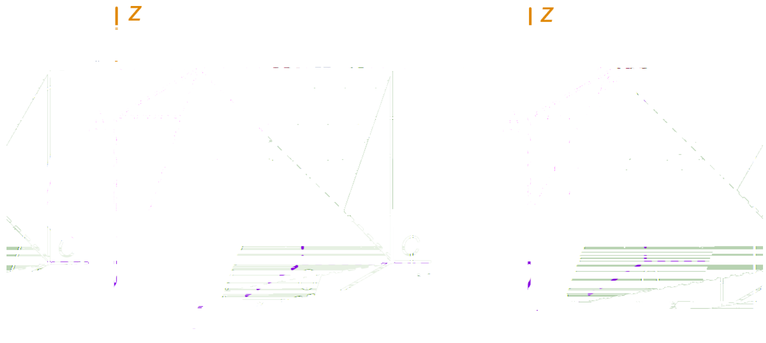


$BCHE$ $\vec{g}=\begin{pmatrix}0\\1\\1\end{pmatrix}$

$ABHG$ $\vec{e}=$ $\begin{pmatrix}1\\0\\1\end{pmatrix}$ $\cos(\;)=$ $\frac{\vec{g}\cdot\vec{e}}{\sqrt{2}\cdot\sqrt{2}}=\frac{1}{2}$ $=60^\circ$

58

ABHG *ACH*



a

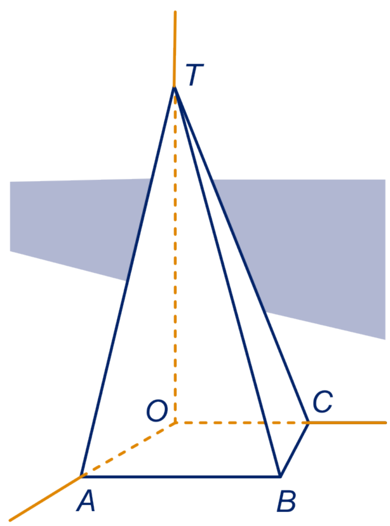
ABHGABHG *HG*
M *EA*

b

Opmerking

59

$ABCO.T$ $A(6,0,0)$ $B(6,6,0)$ $C(0,4,0)$
$T(0,0,10)$

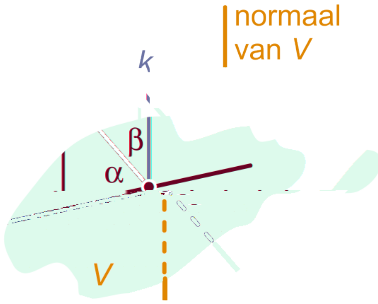


a
ABC *BTC*
BT *OC*

b
BT *ABC*

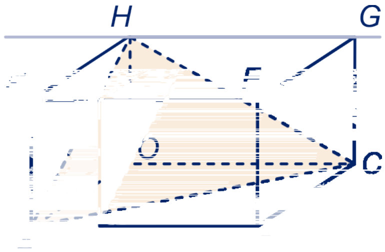
c *TAB* *TBC*

m k V
k V k
$+ \quad = 90^\circ$



V n k
V 90°

Voorbeeld

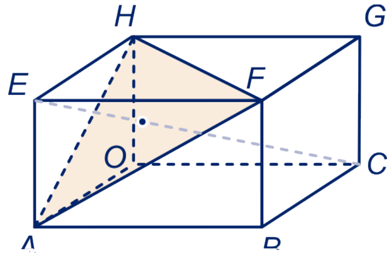


$ABCO.EFGH$
$A(7,0,0)$ $C(0,7,0)$ $H(0,0,4)$ GH
ACH
ACH $\frac{x}{7} + \frac{y}{7} + \frac{z}{4} = 1$

$$\vec{n} = \begin{pmatrix} 4 \\ 4 \\ 7 \end{pmatrix}$$

GH $\vec{y} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ GH

ACH
$\left|\vec{n} \cdot \vec{y}\right| = \left|\vec{n}\right| \cdot \left|\vec{y}\right| \cdot \cos(\ )$ $= 64^\circ$
GH ACH $90^\circ -$ $= 26^\circ$



60 EC AFH

61 $A(6,0,0)$ $B(4,3,2)$ $C(0,0,6)$

a OAB OBC

b BC OAB

c OAB OBC

62 $OABC$

a C OAB

AOB

b $\sin(\ ) = \sqrt{\frac{13}{29}}$

OAB $3\sqrt{13}$

c

d $OABC$

Definitie

$\left(\vec{a}, \vec{b}\right)$

$\vec{a}=\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ $\vec{b}=\begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$

vector $\vec{a} \times \vec{b}$ $\begin{pmatrix} a_2 b_3 - a_3 b_2 \\ a_3 b_1 - a_1 b_3 \\ a_1 b_2 - a_2 b_1 \end{pmatrix}$

$\vec{a} \times \vec{b}$ **uitproduct** $\vec{a}$ $\vec{b}$

63 $\vec{a}=\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ $\vec{b}=\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$ $\vec{c}=\begin{pmatrix} 3 \\ -1 \\ -1 \end{pmatrix}$

$\vec{a} \times \vec{a}$ $\vec{a} \times \vec{b}$ $\vec{b} \times \vec{a}$ $\vec{b} \times \vec{c}$ $\vec{c} \times \vec{a}$

64 $\vec{a}$ $\vec{b}$ $\vec{c}$

a $\left(2\vec{a}\right) \times \vec{b}$ $\vec{a} \times \left(2\vec{b}\right)$ $2\left(\vec{a} \times \vec{b}\right)$

b $\vec{a} \times \vec{c}$ $\vec{b} \times \vec{c}$ $\left(\vec{a} + \vec{b}\right) \times \vec{c}$

c $\vec{a} \cdot \left(\vec{a} \times \vec{b}\right)$ $\vec{b} \cdot \left(\vec{a} \times \vec{b}\right)$

- $\vec{a} \times \vec{b} = -\vec{b} \times \vec{a}$ $\vec{a} \times \vec{a} = \vec{0}$
- $\left(\vec{a} + \vec{b}\right) \times \vec{c} = \vec{a} \times \vec{c} + \vec{b} \times \vec{c}$
- $\vec{a} \times \vec{b}$ $\vec{a}$ $\vec{b}$

$\vec{a}=\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ $\vec{b}=\begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$

$$\vec{a} \cdot \left(\vec{a} \times \vec{b}\right) = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \cdot \begin{pmatrix} a_2 b_3 - a_3 b_2 \\ a_3 b_1 - a_1 b_3 \\ a_1 b_2 - a_2 b_1 \end{pmatrix} =$$

$a_1\left(a_2b_3 - a_3b_2\right) + a_2\cdot\left(a_3b_1 - a_1b_3\right) + a_3\cdot\left(a_1b_2 - a_2b_1\right) = 0$

A B C

$\overrightarrow{AB}\times\overrightarrow{AC}$ ABC

65

a

$A(1,2,3)$ $B(2,3,5)$ $C(3,1,6)$

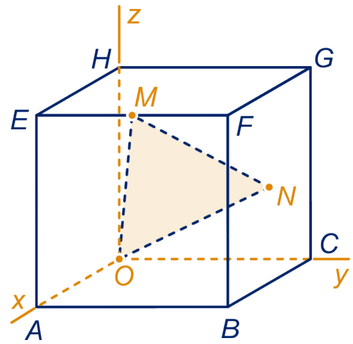
b $\overrightarrow{AB}\times\overrightarrow{AC}$

c ABC

66

2 M

N



a $\vec{n}=\overrightarrow{OM}\times\overrightarrow{ON}$ $\left|\vec{n}\right|$

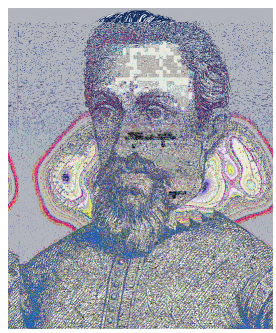
$\overrightarrow{OM}$ $\overrightarrow{ON}$

b $\cos(\ )$ $\sin(\ )$

c

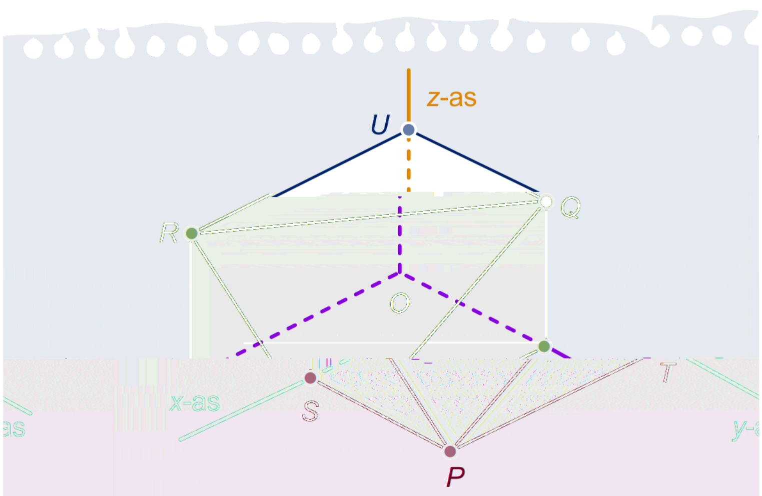
O M N

67



$OSRU$ $OTQU$

$P(6,4,0)$ $Q(0,4,3)$ $R(6,0,3)$



a $\overrightarrow{PQ}\times\overrightarrow{PR}$

b $\overrightarrow{PQ}\times\overrightarrow{PR}$

$\overrightarrow{PQ}$ $\overrightarrow{PR}$

c $\cos(\ )$ $\sin(\ )$

d

P Q R

e PQR

P Q R

$\overrightarrow{PQ} \times \overrightarrow{PR}$

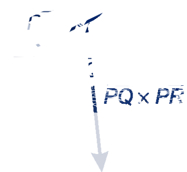
P Q R

De wetten van Kepler

$\overrightarrow{PQ} \times \overrightarrow{PR}$

kurkentrekkerregel $\overrightarrow{PQ} \times \overrightarrow{PR}$

$\overrightarrow{PQ}$ $\overrightarrow{PR}$



Definitie

$\left(\vec{p}, \vec{q}, \vec{r}\right)$ **positief**

geörienteerd

$\vec{p}$ $\vec{q}$ $\vec{r}$ **negatief**

geörienteerd

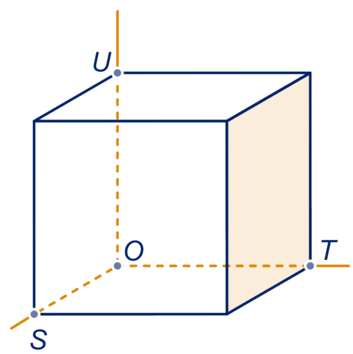
- $\vec{p} \times \vec{q} = -\vec{q} \times \vec{p}$ $\vec{p} \times \vec{p} = \vec{0}$
- $\left(\vec{p} + \vec{q}\right) \times \vec{r} = \vec{p} \times \vec{r} + \vec{q} \times \vec{r}$
- $\vec{p} \times \vec{q}$ $\vec{p}$ $\vec{q}$
- $\left(\vec{p}, \vec{q}, \vec{p} \times \vec{q}\right)$
- $\vec{p} \times \vec{q}$

$\vec{p}$ $\vec{q}$

68

$\vec{p} = \begin{pmatrix}1\\0\\0\end{pmatrix}$ $\vec{q} = \begin{pmatrix}0\\1\\0\end{pmatrix}$ $\vec{r} = \begin{pmatrix}0\\0\\1\end{pmatrix}$

a $\vec{p} \times \vec{q}$ $\vec{q} \times \vec{r}$ $\vec{r} \times \vec{p}$



$O(0,0,0)$ $S(6,0,0)$

$T(0,5,0)$ $U(0,0,4)$

b $\left(\vec{s} \times \vec{t}\right) \cdot \vec{u}$

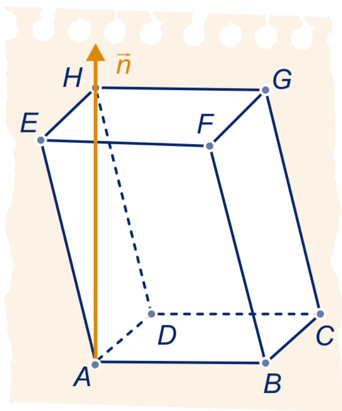
69 $ABCO.EFGH$ $A(5,0,0)$
$C(2,3,0)$ $H(0,-2,6)$

a

b

c $\left(\vec{a} \times \vec{c}\right) \cdot \vec{h}$

$ABCD.EFGH$
$\left(\overrightarrow{AB} \times \overrightarrow{AD}\right) \cdot \overrightarrow{AE}$



$\vec{n}$ $ABCD$

$\dfrac{\left|\overrightarrow{AE} \cdot \vec{n}\right|}{\left|\vec{n}\right|}$ $\overrightarrow{AE}$

A $\vec{n}$

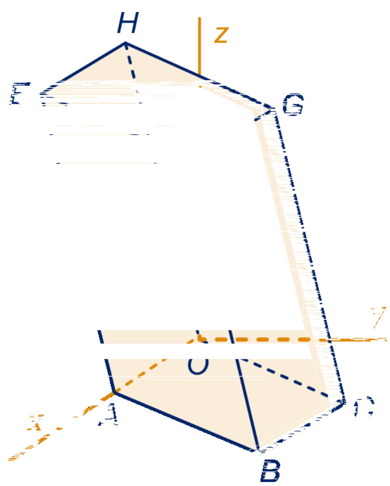
$ABCD$

$\vec{n} = \overrightarrow{AB} \times \overrightarrow{AD}$ $\left|\vec{n}\right|$

$ABCD$ $\dfrac{\left|\overrightarrow{AE} \cdot \vec{n}\right|}{\left|\vec{n}\right|} \left|\vec{n}\right| = \left|\left(\overrightarrow{AB} \times \overrightarrow{AD}\right) \cdot \overrightarrow{AE}\right|$

$ABCD.EFGH$

70 $ABCO.EFGH$
$A(3,0,0)$ $C(6,5,0)$ $H(-3,-3,5)$



a

b

c $ABC.EFG$

d $ABCF$

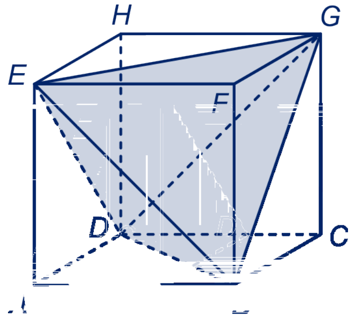
71

$ABCD.EFGH$ 1

$BDEG$

a

$$\frac{1}{6}\left(\begin{pmatrix}1\\1\\0\end{pmatrix}\times\begin{pmatrix}0\\1\\1\end{pmatrix}\right)\cdot\begin{pmatrix}1\\0\\1\end{pmatrix}$$



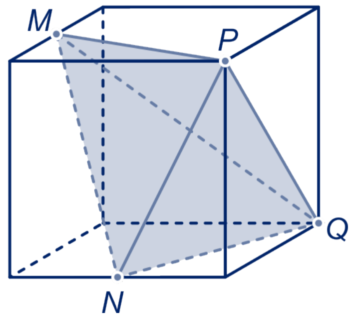
b

72

2

M N P Q

$MNPQ$

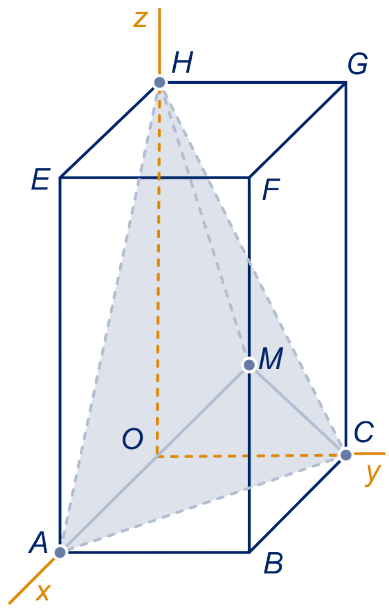


73

$ABCO.EFGH$ 4 3 2

M BF

$ACHM$



De determinant in dimensie 2

De kracht van vectoren

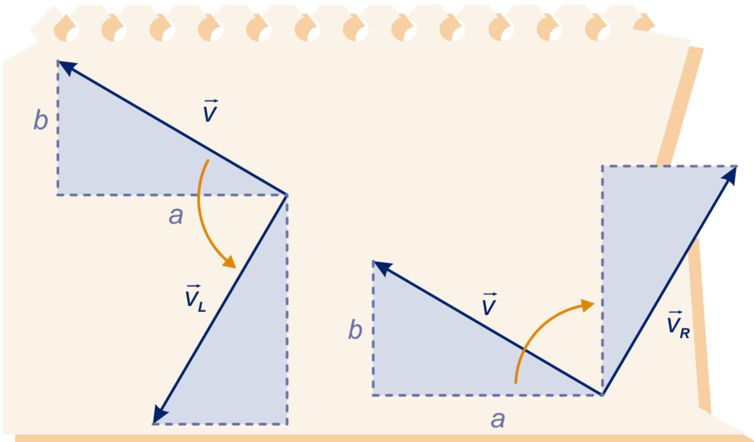
$\vec{v} = \begin{pmatrix} a \\ b \end{pmatrix}$ $\vec{v}_L = \begin{pmatrix} -b \\ a \end{pmatrix}$

$\vec{v}_R = \begin{pmatrix} b \\ -a \end{pmatrix}$ $\vec{v}$

$\vec{v}$ 90° $\vec{v}_L$

$\vec{v}$ 90°

$\vec{v}_R$



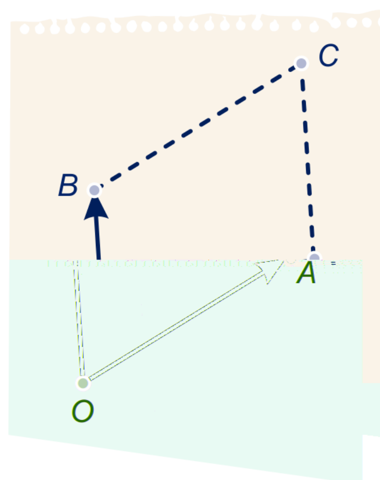
a b

$\vec{a}$ $\vec{b}$ **parallellogram**

opgespannen door $\vec{a}$ $\vec{b}$

$OABC$ C $\vec{a} + \vec{b}$

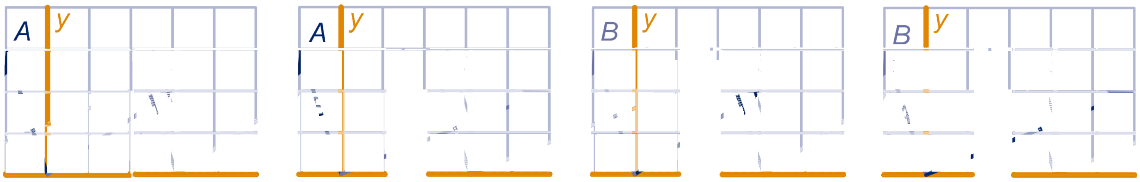
$\vec{a}$ $\vec{b}$



74

A *B*

A *B*



a

$\vec{a}$ $\vec{b}$

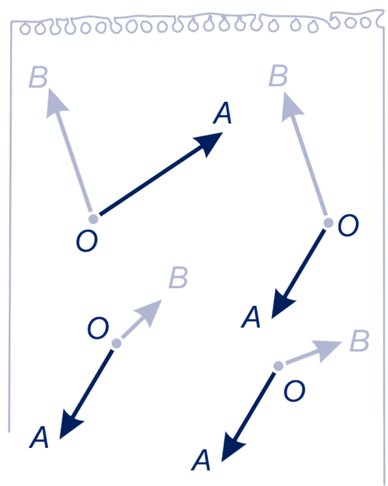
b $\vec{a} \cdot \vec{b}_R$

$\vec{a}$ $\vec{b}$ $\vec{a}$

$\vec{b}_R$

75

$\vec{a}$ $\vec{b}$



$\vec{a}$ $\vec{b}_R$

$\vec{a}$ $\vec{b}$

$0° <\quad < 90°$

$90° <\quad < 180°$

a

$p \cdot q \cdot \sin(\ )$

p q

$\vec{a}$

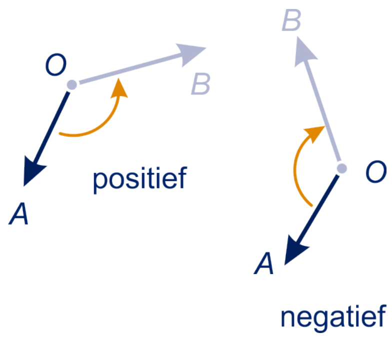
$\vec{b}$ $\vec{a} \cdot \vec{b}_R$ $0° <\quad < 90°$ $-\vec{a} \cdot \vec{b}_R$ $90° <\quad < 180°$

b

$\vec{a} = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$ $\vec{b} = \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}$ $\vec{a} \cdot \vec{b}_R = a_1 \cdot b_2 - a_2 \cdot b_1$

c

$\vec{a} = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$ $\vec{b} = \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}$



$\vec{a}$ O $\vec{b}$

linksom $\left(\vec{a}, \vec{b}\right)$

positief georiënteerd

rechtsom **negatief georiënteerd**

determinant $\left(\vec{a}, \vec{b}\right)$

georiënteerde oppervlakte

$\left(\vec{a}, \vec{b}\right)$

- $\left(\vec{a}, \vec{b}\right)$
- $\left(\vec{a}, \vec{b}\right)$

$\left(\vec{a}, \vec{b}\right)$ **determinant** $\left(\vec{a}, \vec{b}\right)$

Stelling 1

$\left(\vec{a}, \vec{b}\right) = a_1 \cdot b_2 - a_2 \cdot b_1$

Opmerking

$\left(\vec{a}, \vec{b}\right)$ $\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$

roosterdriehoek

76

a

$A(4,7)$ $B(10,3)$ $C(8,10)$

b ABC

2

c

$\frac{1}{2}$

6.6 Determinant en uitproduct

77

a $\begin{vmatrix} 1 & 2 \\ -3 & 4 \end{vmatrix}$ $\begin{vmatrix} 2 & 2 \\ -3 & 4 \end{vmatrix}$ $\begin{vmatrix} -2 & 2 \\ 3 & 4 \end{vmatrix}$ $\begin{vmatrix} 1 & -2 \\ -3 & 6 \end{vmatrix}$

b -2 $\begin{vmatrix} -2 & 2 \\ 6 & 4 \end{vmatrix}$

c $\begin{vmatrix} 1+2 & 2 \\ -3+-3 & 4 \end{vmatrix} = \begin{vmatrix} 3 & 2 \\ -6 & 4 \end{vmatrix}$

d $\left(\vec{a}, \vec{b}\right) = 20$ $\left(\vec{b}, \vec{a}\right)$

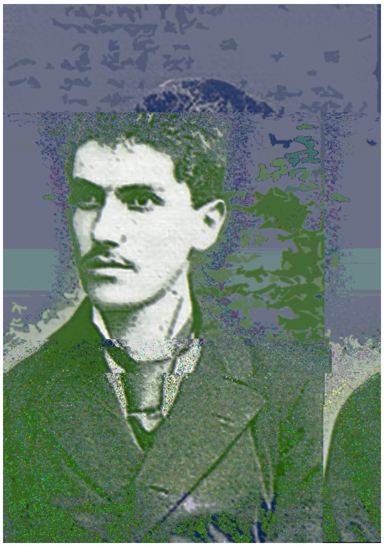
78

a $1\frac{1}{2}$

b 1

r

i $i + \frac{1}{2}r - 1$



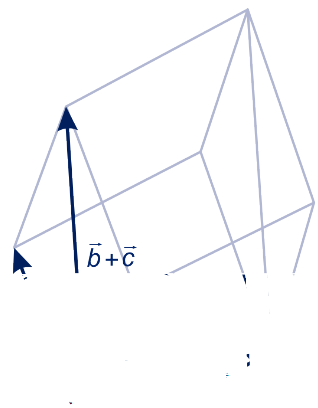
Sommige wiskundige stellingen zijn zo fantastisch simpel en elegant, dat je je afvraagt: "Waarom ben ik daar niet op gekomen!" Dit stukje gaat over precies zo'n stelling: eenvoudiger dan de stelling van Pythagoras, maar onbekend zelfs bij veel professionele wiskundigen. De stelling wordt vernoemd naar haar 'ontdekker': de Oostenrijkse wiskundige Georg Alexander Pick, geboren in 1859 in Wenen en omgekomen in 1942 in het concentratiekamp Theresienstadt, waarheen hij op 82–jarige leeftijd om zijn joodse afkomst gedeporteerd werd.

Eigenschappen van de determinant

$$\left(\vec{a},\vec{b}\right) = - \quad \left(\vec{b},\vec{a}\right)$$
$$\left(\vec{a}+\vec{b},\vec{c}\right) = \quad \left(\vec{a},\vec{b}\right) + \quad \left(\vec{a},\vec{c}\right)$$
$$\left(\vec{a},\vec{b}+\vec{c}\right) = \quad \left(\vec{a},\vec{b}\right) + \quad \left(\vec{a},\vec{c}\right)$$
$$\left(k\cdot\vec{a},\vec{b}\right) = \quad \left(\vec{a},k\cdot\vec{b}\right) = k\cdot \quad \left(\vec{a},\vec{b}\right)$$
$$\left(\vec{a},\vec{b}\right) = 0 \Leftrightarrow \left(\vec{a},\vec{b}\right)$$

79

$\vec{a}\ \vec{b}\ \vec{c} \quad \vec{b} + \vec{c}$



a $\left(\vec{a},\vec{b}+\vec{c}\right) =$

$\left(\vec{a},\vec{b}\right) + \quad \left(\vec{a},\vec{c}\right)$

b $\left(2\cdot\vec{a},\vec{b}\right) = 2\cdot$

$\left(\vec{a},\vec{b}\right)$

80

$$\vec{e}_1 = \begin{pmatrix}1\\0\end{pmatrix} \quad \vec{e}_2 = \begin{pmatrix}0\\1\end{pmatrix}$$

$\begin{pmatrix}a_1\\a_2\end{pmatrix} = a_1\cdot\vec{e}_1 + a_2\cdot\vec{e}_2 \quad \begin{pmatrix}b_1\\b_2\end{pmatrix} = b_1\cdot\vec{e}_1 + b_2\cdot\vec{e}_2$

$$\left(a_1\cdot\vec{e}_1 + a_2\cdot\vec{e}_2, b_1\cdot\vec{e}_1 + b_2\cdot\vec{e}_2\right) =$$

$a_1\cdot b_1\cdot \quad \left(\vec{e}_1,\vec{e}_1\right) + a_1\cdot b_2\cdot \quad \left(\vec{e}_1,\vec{e}_2\right) + a_2\cdot b_1\cdot \quad \left(\vec{e}_2,\vec{e}_1\right) +$

$a_2\cdot b_2\cdot \quad \left(\vec{e}_2,\vec{e}_2\right)$

a

$$\left(\vec{e}_1,\vec{e}_1\right) = 0 \quad \left(\vec{e}_1,\vec{e}_2\right) = 1 \quad \left(\vec{e}_2,\vec{e}_1\right) = -1$$
$$\left(\vec{e}_2,\vec{e}_2\right) = 0$$

b

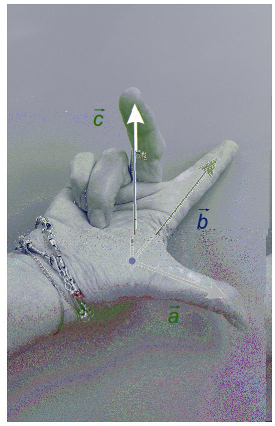
$$\left(\vec{a},\vec{b}\right) = a_1\cdot b_2 - a_2\cdot b_1$$

81 k $y = 2x + 7$
$A(-3,7)$ $B(5,5)$
k C ABC
15
C

De determinant in dimensie 3.

Definitie

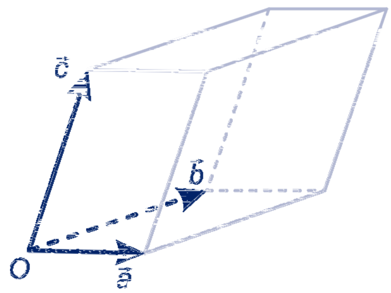
$\left(\vec{a}, \vec{b}, \vec{c}\right)$

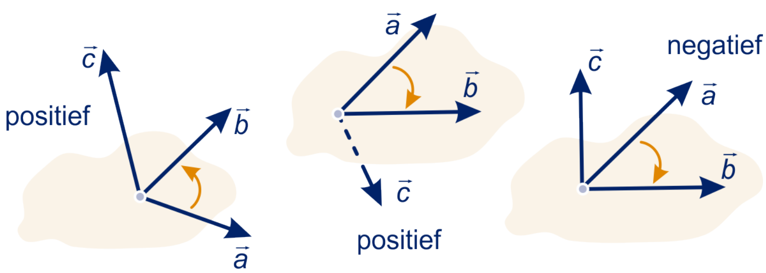


$\vec{a}$
$\vec{b}$ *rechterhand*
$\vec{c}$ **positief georiënteerd**

$\vec{a}$ $\vec{b}$ $\vec{c}$
parallellepipedum opgespannen

determinant $\left(\vec{a}, \vec{b}, \vec{c}\right)$
$\left(\vec{a}, \vec{b}, \vec{c}\right)$
$\vec{a}$ $\vec{b}$ $\vec{c}$
$\left(\vec{a}, \vec{b}, \vec{c}\right) = 0$





82

$\left(\vec{a}, \vec{b}, \vec{c}\right)$ 6

$\left(\vec{a}, \vec{b}, \vec{c}\right)$

Eigenschappen van de determinant in dimensie 3

$\left(\vec{a}, \vec{b}, \vec{c}\right) = \left(\vec{b}, \vec{c}, \vec{a}\right) = \left(\vec{c}, \vec{a}, \vec{b}\right) =$

$\left(\vec{b}, \vec{a}, \vec{c}\right) = \left(\vec{a}, \vec{c}, \vec{b}\right) = \left(\vec{c}, \vec{b}, \vec{a}\right)$

$\left(\vec{a} + \vec{d}, \vec{b}, \vec{c}\right) = \left(\vec{a}, \vec{b}, \vec{c}\right) + \left(\vec{d}, \vec{b}, \vec{c}\right)$

$\left(\vec{a}, \vec{b} + \vec{d}, \vec{c}\right) = \left(\vec{a}, \vec{b}, \vec{c}\right) + \left(\vec{a}, \vec{d}, \vec{c}\right)$

$\left(\vec{a}, \vec{b}, \vec{c} + \vec{d}\right) = \left(\vec{a}, \vec{b}, \vec{c}\right) + \left(\vec{a}, \vec{b}, \vec{d}\right)$

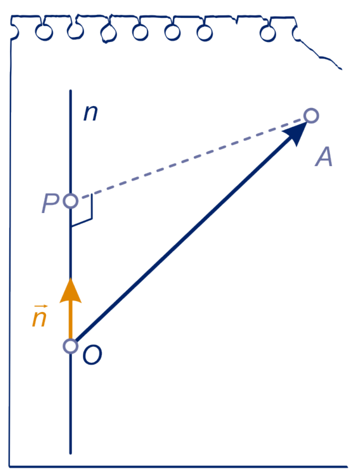
$\left(k \cdot \vec{a}, \vec{b}, \vec{c}\right) = \left(\vec{a}, k \cdot \vec{b}, \vec{c}\right) = \left(\vec{a}, \vec{b}, k \cdot \vec{c}\right) =$

$k \cdot \left(\vec{a}, \vec{b}, \vec{c}\right)$

$\left(\vec{a}, \vec{b}, \vec{c}\right) = 0 \Leftrightarrow \left(\vec{a}, \vec{b}, \vec{c}\right)$

$\left(\vec{a}, \vec{b}, \vec{c}\right)$

Stelling 2



n O $\vec{n}$

n 1

$\vec{a}$ n

$\left(\vec{a} \cdot \vec{n}\right) \cdot \vec{n}$

$\vec{a}$ $P\left(\vec{a}\right)$

P **lineair**

$P\left(\vec{a} + \vec{b}\right) = P\left(\vec{a}\right) + P\left(\vec{b}\right)$

$P\left(k \cdot \vec{a} + \vec{b}\right) = k \cdot P\left(\vec{a}\right)$

Bewijs

$\vec{n}$ 1 $\left(\vec{a} \cdot \vec{n}\right) \cdot \vec{n}$

$$\vec{a} \cdot \vec{n}$$

$$\left|\vec{a}\right| \cdot \left|\vec{n}\right| \cdot \cos(\ ) = \left|\vec{a}\right| \cdot \cos(\ )$$

$$\vec{a} \quad \vec{n} \qquad \left(\vec{a} \cdot \vec{n}\right) \cdot \vec{n}$$

P

$$\left(\vec{a} + \vec{b}\right) \cdot \vec{n} = \vec{a} \cdot \vec{n} + \vec{b} \cdot \vec{n} \qquad \left(k \cdot \vec{a}\right) \cdot \vec{n} = k \cdot \left(\vec{a} \cdot \vec{n}\right)$$

Bewijs van de eigenschappen

$\vec{a}\ \vec{b}\ \vec{c}$ $\vec{d}$ V

$\vec{a}$ $\vec{b}$ $\vec{n}$ V

1 $\left(\vec{a}, \vec{b}, \vec{n}\right)$

Opp

$\vec{a}$ $\vec{b}$

$$\left(\vec{a}, \vec{b}, \vec{v}\right) = \pm Opp \cdot \left|P\left(\vec{v}\right)\right|$$

$\left(\vec{a}, \vec{b}, \vec{v}\right)$ $\left(\vec{a}, \vec{b}, \vec{c}\right)$

$\left(\vec{a}, \vec{b}, \vec{d}\right)$ $\left(\vec{a}, \vec{b}, \vec{c} + \vec{d}\right) =$

$Opp \cdot \left|P\left(\vec{c} + \vec{d}\right)\right| = Opp \cdot (|P\left(\vec{c}\right) + P\left(\vec{d}\right)|) =$

$Opp \cdot (|P\left(\vec{c}\right)| + |P\left(\vec{d}\right)|) = \left(\vec{a}, \vec{b}, \vec{c}\right) + \left(\vec{a}, \vec{b}, \vec{d}\right)$

P

$$\vec{e}_1 = \begin{pmatrix}1\\0\\0\end{pmatrix} \quad \vec{e}_2 = \begin{pmatrix}0\\1\\0\end{pmatrix} \quad \vec{e}_3 = \begin{pmatrix}0\\0\\1\end{pmatrix}$$

$$\begin{pmatrix}x\\y\\z\end{pmatrix} = x \cdot \vec{e}_1 + y \cdot \vec{e}_2 + z \cdot \vec{e}_3$$

$$\vec{a} = \begin{pmatrix}a_1\\a_2\\a_3\end{pmatrix} \quad \vec{b} = \begin{pmatrix}b_1\\b_2\\b_3\end{pmatrix} \quad \vec{c} = \begin{pmatrix}c_1\\c_2\\c_3\end{pmatrix}$$

$\left(\vec{a}, \vec{b}, \vec{c}\right)$ 27

83

$a_1 \cdot b_1 \cdot c_2 \cdot$ $\left(\vec{e}_1, \vec{e}_1, \vec{e}_2\right)$ $a_3 \cdot b_2 \cdot c_1 \cdot$ $\left(\vec{e}_3, \vec{e}_2, \vec{e}_1\right)$

a

27 $\left(\vec{e}_1, \vec{e}_1, \vec{e}_2\right)$ $\left(\vec{e}_3, \vec{e}_2, \vec{e}_1\right)$

6 0

b

$$\left(\vec{a}, \vec{b}, \vec{c}\right) =$$

$a_1 \cdot b_2 \cdot c_3 + a_2 \cdot b_3 \cdot c_1 + a_3 \cdot b_1 \cdot c_2 +$
$-a_3 \cdot b_2 \cdot c_1 - a_2 \cdot b_1 \cdot c_3 - a_1 \cdot b_3 \cdot c_2$

c

$$\vec{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \quad \vec{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} \quad \vec{c} = \begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix}$$

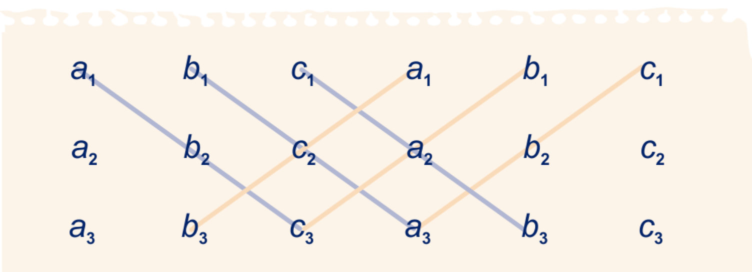
$$\left(\vec{a}, \vec{b}, \vec{c}\right) =$$

$a_1 \cdot b_2 \cdot c_3 + a_2 \cdot b_3 \cdot c_1 + a_3 \cdot b_1 \cdot c_2 +$
$-a_3 \cdot b_2 \cdot c_1 - a_2 \cdot b_1 \cdot c_3 - a_1 \cdot b_3 \cdot c_2$

$$\left(\vec{a}, \vec{b}, \vec{c}\right) \qquad \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

Opmerking

$\vec{a}, \vec{b}, \vec{c}$



B

G $B - G$

Voorbeeld

$$\begin{vmatrix} 1 & -4 & 7 \\ 2 & -5 & 8 \\ 3 & -6 & 9 \end{vmatrix} =$$

$1 \cdot -5 \cdot 9 + -4 \cdot 8 \cdot 3 + 7 \cdot 2 \cdot -6 - (-6 \cdot 8 \cdot 1 + 9 \cdot 2 \cdot -4 + 3 \cdot -5 \cdot 7) =$

0

Voorbeeld

$$\begin{vmatrix} 1 & 0 & -1 \\ 1 & 1 & 0 \\ 0 & 1 & -1 \end{vmatrix} = -1 + 0 + -1 - (0 + 0 + 0) = -2$$

84

$$\begin{vmatrix} 1 & 0 & 3 \\ 1 & 1 & 0 \\ 1 & 2 & -1 \end{vmatrix} \quad \begin{vmatrix} 3 & 2 & 3 \\ 1 & 1 & 4 \\ 1 & 2 & -1 \end{vmatrix} \quad \begin{vmatrix} 3 & 12 & 3 \\ -1 & 1 & 0 \\ 0 & 12 & -1 \end{vmatrix}$$

85

$ABCD.EFGH$

$A(2,0,0)$ $B(4,2,1)$ $C(5,-1,0)$ $E(2,0,10)$

a H

40

b

c $ABDE$

$EFCG$

86

$\frac{2}{3}$

87

$\vec{r} = \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}$ $\vec{s} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$

a b

$A(a, b, 2)$ $\vec{x} = p \cdot \vec{r} + q \cdot \vec{s}$

$$\begin{vmatrix} a & 2 & 1 \\ b & 0 & 1 \\ 2 & 1 & 1 \end{vmatrix} = 0$$

a

b a b

V $\vec{x}=\vec{c}+p\cdot\vec{r}+q\cdot\vec{s}$ $\vec{c}=\begin{pmatrix}3\\0\\0\end{pmatrix}$

c a b A V

88

$$\begin{vmatrix}1 & 4 & x\\2 & -1 & y\\3 & 0 & z\end{vmatrix}=0$$

a

O V

b (1,2,3) (4,-1,0) V

$$\vec{a}=\begin{pmatrix}a_1\\a_2\\a_3\end{pmatrix} \quad \vec{b}=\begin{pmatrix}b_1\\b_2\\b_3\end{pmatrix}$$

$$\begin{vmatrix}a_1 & b_1 & x\\a_2 & b_2 & y\\a_3 & b_3 & z\end{vmatrix} = 0$$

$\vec{a}$ $\vec{b}$

$\left(\vec{a},\vec{b},\vec{a}\right)=0$ $\left(\vec{a},\vec{b},\vec{b}\right)=0$

89

$$\begin{vmatrix}a_1 & b_1 & x\\a_2 & b_2 & y\\a_3 & b_3 & z\end{vmatrix}=\begin{vmatrix}a_2 b_2\\a_3 b_3\end{vmatrix}\cdot x-\begin{vmatrix}a_1 b_1\\a_3 b_3\end{vmatrix}\cdot y+\begin{vmatrix}a_1 b_1\\a_2 b_2\end{vmatrix}\cdot z$$

Definitie

$\left(\vec{a},\vec{b}\right)$

$$\vec{a}=\begin{pmatrix}a_1\\a_2\\a_3\end{pmatrix} \quad \vec{b}=\begin{pmatrix}b_1\\b_2\\b_3\end{pmatrix}$$

vector $\vec{a}\times\vec{b}$ $\begin{pmatrix}a_2b_3-a_3b_2\\a_3b_1-a_1b_3\\a_1b_2-a_2b_1\end{pmatrix}$

$\vec{a}\times\vec{b}$ **uitproduct** $\vec{a}$ $\vec{b}$

$$\left(\vec{a},\vec{b},\vec{x}\right)=\left(\vec{a}\times\vec{b}\right)\cdot\vec{x}$$

$\vec{a}\times\vec{b}$ $\vec{a}$ $\vec{b}$

$\left|\vec{a}\times\vec{b}\right|$

$\vec{a}$ $\vec{b}$

$\left(\vec{a}\times\vec{b}\right)\cdot\vec{a}=0$ $\left(\vec{a}\times\vec{b}\right)\cdot\vec{b}=0$ $\left(\vec{a},\vec{b},\vec{a}\right)=0$

$\left(\vec{a},\vec{b},\vec{b}\right)=0$

$\vec{a}\times\vec{b}$ $\vec{a}$ $\vec{b}$

$\vec{n}$

$\vec{a}$ $\vec{b}$ 1 $\left(\vec{a},\vec{b},\vec{n}\right)$

$\left(\vec{a},\vec{b},\vec{n}\right)$

$\vec{a}$ $\vec{b}$

$\vec{a}\times\vec{b}$ $\vec{n}$ $\vec{a}\times\vec{b}$

Voorbeeld

V

$(x,y,z)=(2,1,3)+s\cdot(\text{-}2,2,3)+t\cdot(1,2,0)$

V

V

$$\begin{pmatrix}-2\\2\\3\end{pmatrix}\times\begin{pmatrix}1\\2\\0\end{pmatrix}=\begin{pmatrix}2\cdot0-3\cdot2\\3\cdot1-\text{-}2\cdot0\\\text{-}2\cdot2-2\cdot1\end{pmatrix}=\begin{pmatrix}-6\\3\\-6\end{pmatrix}$$

$2x-y+2z=9$

90 $\vec{a}=\begin{pmatrix}1\\2\\3\end{pmatrix}$ $\vec{b}=\begin{pmatrix}1\\-1\\1\end{pmatrix}$ $\vec{c}=\begin{pmatrix}3\\-1\\-1\end{pmatrix}$

$\vec{a}\times\vec{b}$ $\vec{b}\times\vec{c}$ $\vec{c}\times\vec{a}$

91 $ABCD.EFGH$

$A(2,0,0)$ $B(4,2,1)$ $C(5,\text{-}1,0)$ $E(2,0,10)$

- $\vec{a}\times\vec{a}=\vec{0}$
- $\vec{a}\times\vec{b}=-\vec{b}\times\vec{a}$

- $\vec{a} \times (\vec{b} \times \vec{c}) + \vec{b} \times (\vec{c} \times \vec{a}) + \vec{c} \times (\vec{a} \times \vec{b}) = \vec{0}$
- $\vec{a} \times (\vec{b} + \vec{c}) = \vec{a} \times \vec{b} + \vec{a} \times \vec{c}$ $\vec{a} \times (k \cdot \vec{b}) = k \cdot (\vec{a} \times \vec{b})$

92

V $\vec{x} = \vec{p} + s \cdot \vec{a} + t \cdot \vec{b}$ W

$\vec{x} = \vec{q} + s \cdot \vec{c} + t \cdot \vec{d}$

$(\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{d})$

V W

Lorentzkracht

$\vec{i}$ $\vec{B}$

$\vec{F}$

$\vec{F} = c \cdot \vec{i} \times \vec{B}$ c

Opmerking

$\vec{a}$ $\vec{b}$

$\vec{c}$ $(\vec{a}, \vec{b}, \vec{c})$

Lijnen en vlakken

$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} + t \cdot \begin{pmatrix} 0 \\ -4 \\ 7 \end{pmatrix}$ **vectorvoorstelling**

$(1,2,-1)$ $\begin{pmatrix} 0 \\ -4 \\ 7 \end{pmatrix}$

parametervoorstelling

$(x, y, z) = (1, 2 - 4t, -1 + 7t)$

Opmerking

$(a_1 + b_1, a_2 + b_2, a_3 + b_3)$

$(a_1, a_2, a_3) + (b_1, b_2, b_3)$ $(k \cdot a_1, k \cdot a_2, k \cdot a_3)$

$k \cdot (a_1, a_2, a_3)$

$(x, y, z) = (1,2,-1) + t \cdot (0,-4,7)$

$(x, y, z) = (1,2,-1) + (0,-4t,7t)$

kruisend

hoek van twee (kruisende) lijnen

V A V

O V $\vec{r}$

$\vec{s}$ $\vec{x} = \vec{a} + p \cdot \vec{r} + q \cdot \vec{s}$ p q

vectorvoorstelling V

p q

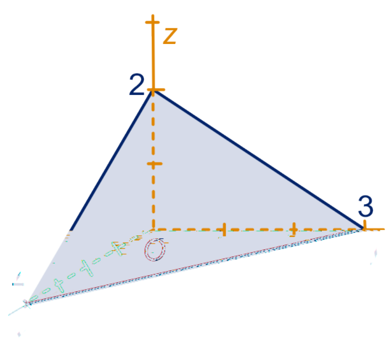
$\vec{x}$ V

Voorbeeld

V $(4,0,0)$ $(0,3,0)$ $(0,0,2)$

O

V $\begin{pmatrix} 4 \\ -3 \\ 0 \end{pmatrix}$ $\begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix}$

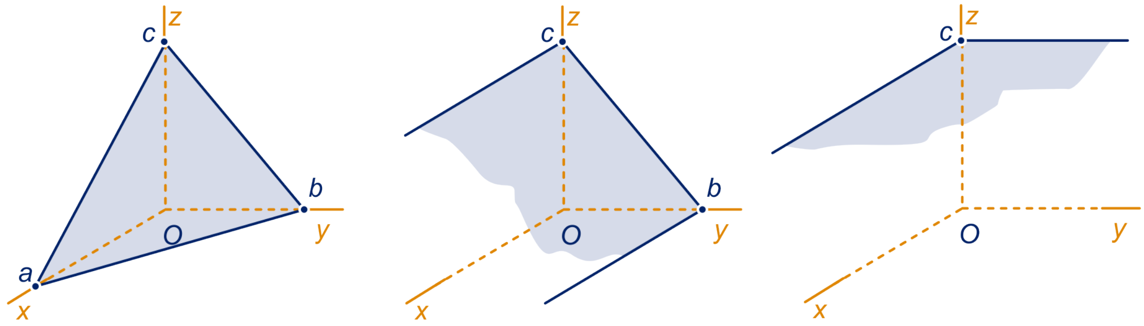


$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 4 \\ 0 \\ 0 \end{pmatrix} + p \cdot \begin{pmatrix} 4 \\ -3 \\ 0 \end{pmatrix} + q \cdot \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix}$$

parametervoorstelling V

$(x, y, z) = (4 + 4p + 2q, -3p, -q)$

Vergelijking van een vlak



a b c 0

- $(a,0,0)$ $(0,b,0)$ $(0,0,c)$ $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$
- x y $(0,b,0)$ z $(0,0,c)$ $\frac{y}{b}+\frac{z}{c}=1$
- Oxy z $(0,0,c)$ $\frac{z}{c}=1$

$ax+by+cz=d$

$$\begin{pmatrix} a \\ b \\ c \end{pmatrix}$$

normaalvector

normaal

Lengte, inproduct, afstand

$X=(x,y,z)$ $\overrightarrow{OX}=\sqrt{x^2+y^2+z^2}$

x y z

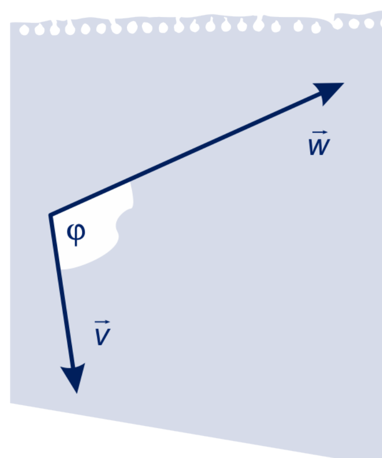
$\vec{v}$ $\left|\vec{v}\right|$

inproduct $\vec{v}=\begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}$ $\vec{w}=\begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix}$ $v_1\cdot w_1+v_2\cdot w_2+v_3\cdot w_3$

$\vec{v}$ $\vec{w}$ $\vec{v}\cdot\vec{w}$

$\vec{v}\cdot\vec{w}=\left|\vec{v}\right|\cdot\left|\vec{w}\right|\cdot\cos(\ )$

$\vec{v}$ $\vec{w}$

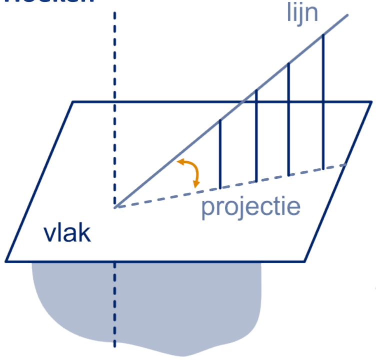


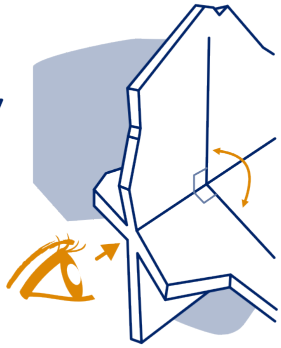
V $n_1 \cdot x + n_2 \cdot y + n_3 \cdot z - d = 0$

P (p_1, p_2, p_3) P V

$$\frac{|n_1 \cdot p_1 + n_2 \cdot p_2 + n_3 \cdot p_3 - d|}{\sqrt{{n_1}^2 + {n_2}^2 + {n_3}^2}}$$

Hoeken





hoek van een lijn met een vlak

$+ \quad = 90^\circ$

hoek van twee vlakken

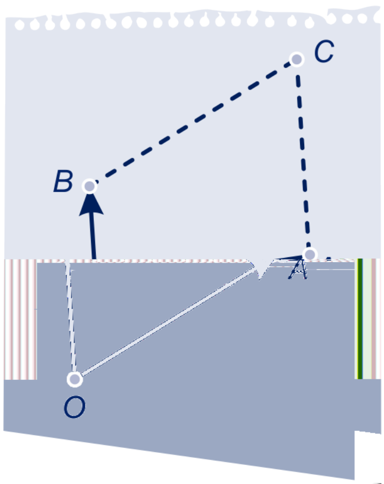
De determinant

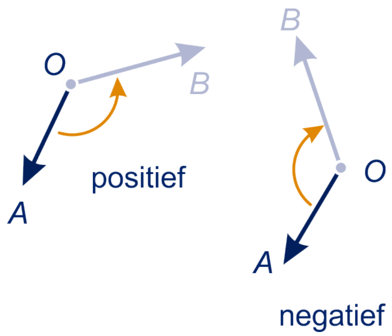
$\vec{a}$ $\vec{b}$

parallellogram opgespannen door $\vec{a}$ $\vec{b}$

$OABC$ C

$\vec{a} + \vec{b}$





De determinant in dimensie 2

$\vec{a} = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$ $\vec{b} = \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}$

$\vec{a}$ O $\vec{b}$

linksom $\left(\vec{a}, \vec{b}\right)$

positief **georiënteerd** **negatief** **georiënteerd**

determinant $\left(\vec{a}, \vec{b}\right)$

georiënteerde **oppervlakte**

$\left(\vec{a}, \vec{b}\right)$

- $\left(\vec{a}, \vec{b}\right)$
- $\left(\vec{a}, \vec{b}\right)$

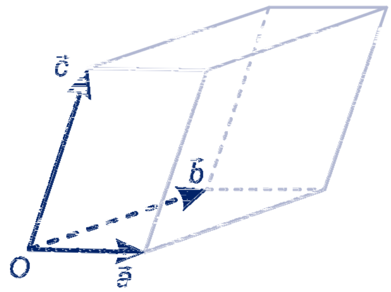
$\left(\vec{a}, \vec{b}\right)$ **determinant** $\left(\vec{a}, \vec{b}\right)$

Stelling

$$\left(\vec{a}, \vec{b}\right) = a_1 \cdot b_2 - a_2 \cdot b_1$$

$\left(\vec{a}, \vec{b}\right)$ $\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$

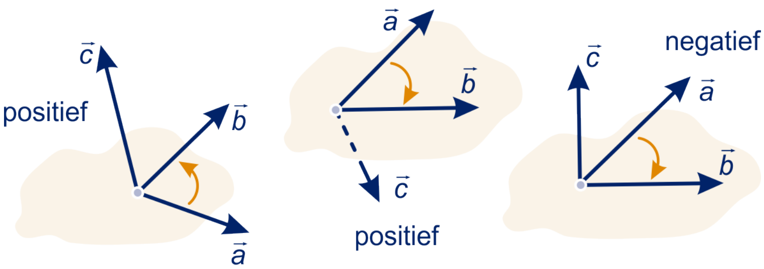
De determinant in dimensie 3 $\left(\vec{a}, \vec{b}, \vec{c}\right)$



$\left(\vec{a}, \vec{b}, \vec{c}\right)$

$\vec{a}$ $\vec{b}$ $\vec{c}$

$\left(\vec{a}, \vec{b}, \vec{c}\right)$ **positief georiënteerd** **negatief georiënteerd**



determinant $\left(\vec{a}, \vec{b}, \vec{c}\right)$

$\left(\vec{a}, \vec{b}, \vec{c}\right)$

$\left(\vec{a}, \vec{b}, \vec{c}\right)$

Stelling

$$\vec{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \vec{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} \quad \vec{c} = \begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix}$$

$$\left(\vec{a}, \vec{b}, \vec{c}\right) =$$

$a_1 \cdot b_2 \cdot c_3 + a_2 \cdot b_3 \cdot c_1 + a_3 \cdot b_1 \cdot c_2 +$
$-a_3 \cdot b_2 \cdot c_1 - a_2 \cdot b_1 \cdot c_3 - a_1 \cdot b_3 \cdot c_2$

$\left(\vec{a}, \vec{b}, \vec{c}\right)$ $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$

$$\left(\vec{a}, \vec{b}, \vec{x}\right) = \left(\vec{a} \times \vec{b}\right) \cdot \vec{x}$$

Het uitproduct

$\left(\vec{a}, \vec{b}\right)$

$\vec{a}=\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ $\vec{b}=\begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$

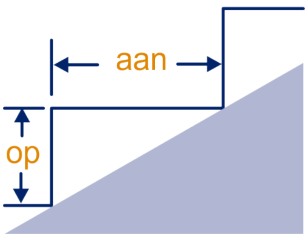
vector $\vec{a} \times \vec{b}$ $\begin{pmatrix} a_2 b_3 - a_3 b_2 \\ a_3 b_1 - a_1 b_3 \\ a_1 b_2 - a_2 b_1 \end{pmatrix}$

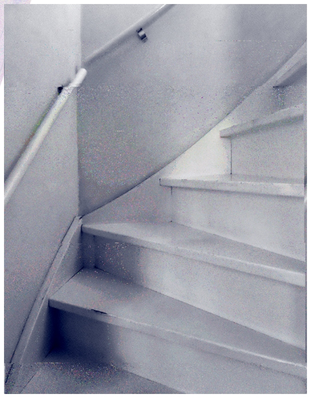
$\vec{a} \times \vec{b}$ **uitproduct** $\vec{a}$ $\vec{b}$

6.8 Extra opgaven

1

$90°$

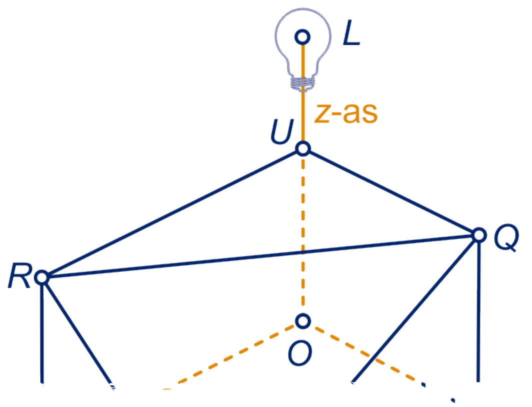




2

$OSRU$ $OTQU$

$P(6,4,0)$ $Q(0,4,3)$ $R(6,0,3)$



$L(0,0,5)$

PQR Oxy

a

b

z

PQR Oxy

c

d $\overrightarrow{PQ} \times \overrightarrow{PR}$

e PQR

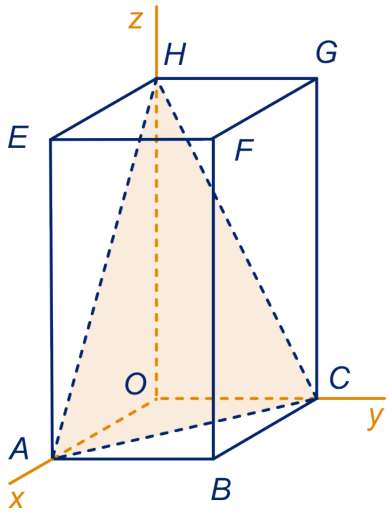
f

3

$ABCO.EFGH$

$A(3,0,0)$ $C(0,2,0)$ $H(0,0,4)$

$\overrightarrow{AH} \times \overrightarrow{CH}$ $\vec{u}$



a $\vec{u}$

b $\vec{u}$

ACH

c $\vec{u}$ EC

AFH

O ACH P

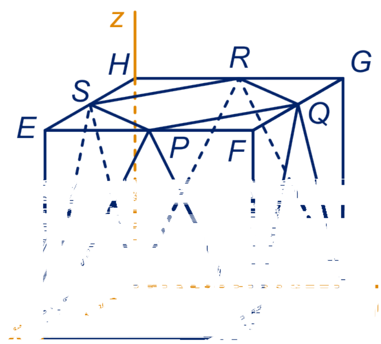
d P

4

$ABCO.EFGH$ $A(6,0,0)$ $C(0,6,0)$

$H(0,0,6)$ P Q R S

$ABCO.PQRS$ L



a L

b L $ABGH$

c

d BPQ

e ABP BPQ

L

f

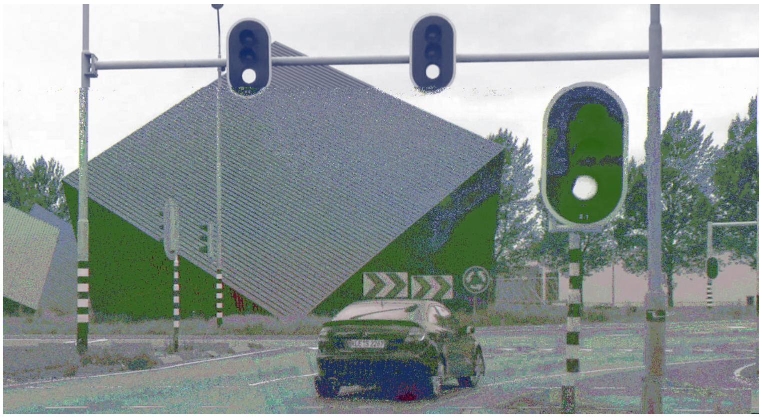
g

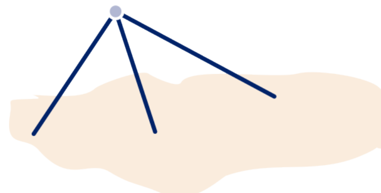
12

6.8 Extra opgaven

5

3 4 5





6

45°

a

60°

b

45°

7

$ABCO.T$ $A(6,0,0)$ $B(6,6,0)$ $C(0,4,0)$
$T(0,0,10)$

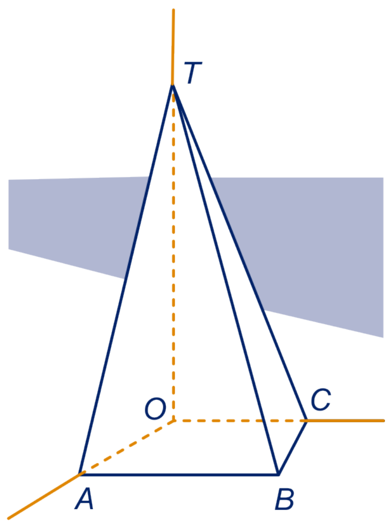
a OAT BCT

TB P OP BT

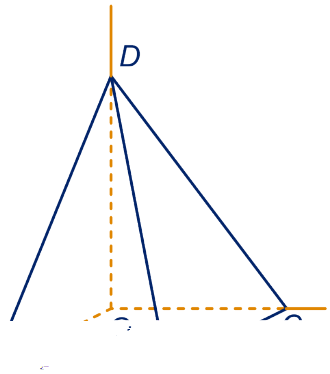
b P

T TBC

c BC



8



$ABCO.T$

$A(3,0,0)$ $C(0,3,0)$ $D(0,0,4)$

a

ABD BCD

ABD BCD

A C

b

c

BD

A DC

d

e

6.8 Extra opgaven

Coördinaten en vectoren

1

a $\overrightarrow{PF}=\begin{pmatrix}6\\4\\6\end{pmatrix}$ $\overrightarrow{AH}=\begin{pmatrix}-6\\0\\6\end{pmatrix}$

b $\overrightarrow{PF}$ $\overrightarrow{AH}$

2

a $C=(-4,4,0)$ $D=(-4,-4,0)$

b $(3,3,2)$

c $\overrightarrow{BT}=\begin{pmatrix}-4\\-4\\8\end{pmatrix}$ $\overrightarrow{BP}=\begin{pmatrix}-1\\-1\\2\end{pmatrix}$

d $\overrightarrow{BR}=\begin{pmatrix}-1\frac{1}{5}\\-1\frac{1}{5}\\2\frac{2}{5}\end{pmatrix}$ $R=\left(2\frac{4}{5},2\frac{4}{5},2\frac{2}{5}\right)$

3

a BT $(4-4t,4-4t,8t)$ y z
$4-4t=8t$ $t=\frac{1}{3}$ $\left(2\frac{2}{3},2\frac{2}{3},2\frac{2}{3}\right)$

b T $\frac{1}{4}\cdot\overrightarrow{BT}$

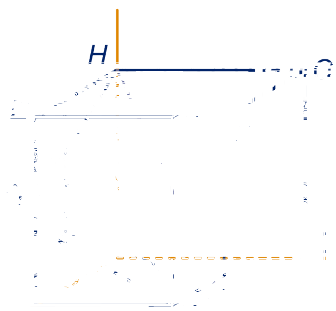
c $(x,y,z)=(-t,t,8+2t)$ $(x,y,z)=(2t,-2t,8-4t)$

4

a AE BP Q

b
R P
BH HQ

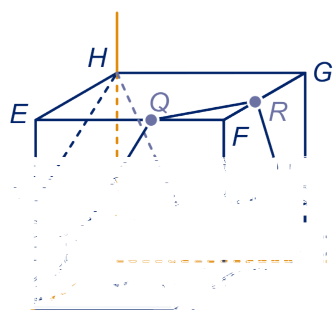
c k $(x,y,z)=(4+t,1+t,2-t)$
R $y=0\Leftrightarrow t=-1$
$(3,0,3)$



d S ACP

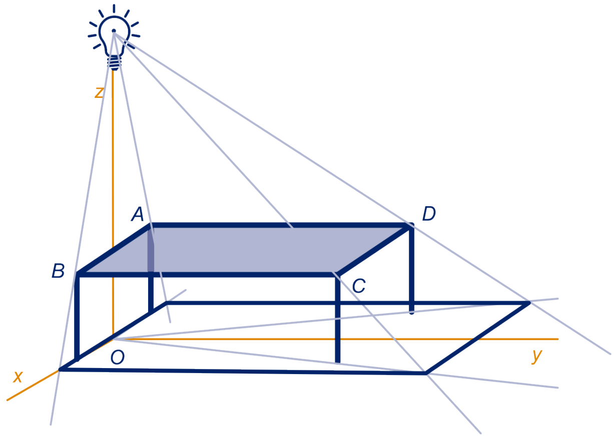
AC
S SP AC

e m $(x,y,z)=(4-t,1+t,2)$
S $t=3$
$(1,4,2)$



5

a



A B C D A' B' C' D'

C' C O

C C

C'

b $\frac{12}{8} = 1\frac{1}{2}$

120 180

c $(x, y, z) = (t, 3t, 12 - 2t)$ LC C

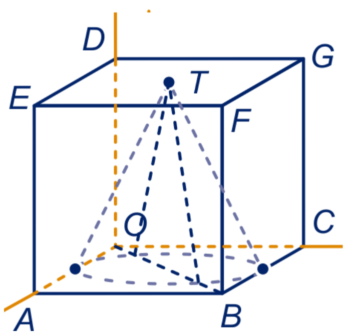
$t = 6$ $(6,18,0)$

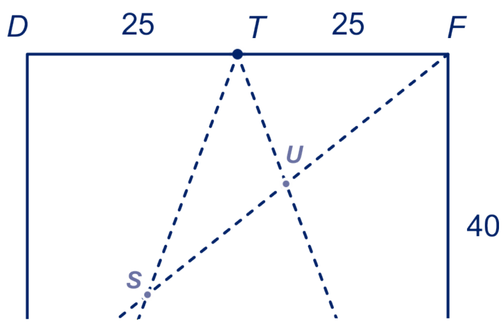
d

$1\frac{1}{2}$ $\left(3, 4\frac{1}{2}, 0\right)$ $(2,3,4)$

6

a





OB P Q $OBFD$

PQT S PT OF U

TQ OF

b

c TSF SOP

$s = 1$ $t = 1\frac{2}{7}$

$a = 1\frac{2}{7}$ $\begin{pmatrix} -28 \\ 7 \\ 9 \end{pmatrix}$

Het inproduct in de ruimte

10

11 **a** AC AH

b 2

AC EG

EGM EGM $2\sqrt{2}$ $\sqrt{5}$

$\sqrt{5}$ EGM $\cos^{-1}\left(\frac{\sqrt{2}}{\sqrt{5}}\right) = 51°$

AB RM

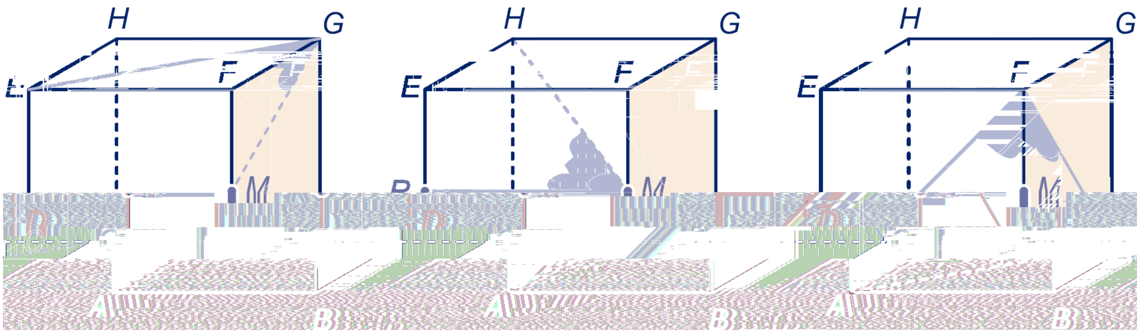
RMH RMH R

2 $\sqrt{5}$ RMH $\tan^{-1}\left(\frac{\sqrt{5}}{2}\right) = 48°$

DE FC

AFC AFC

$60°$



12 **a** $5\sqrt{2}$

b $\sqrt{p^2 + q^2 + r^2}$

c $\overrightarrow{BH} = \begin{pmatrix} -3 \\ -4 \\ 5 \end{pmatrix}$ $5\sqrt{2}$

13 **a** $\overrightarrow{PQ} = \begin{pmatrix} -2 \\ 1 \\ -6 \end{pmatrix}$ $\left|\overrightarrow{PQ}\right| = \sqrt{2^2 + 1^2 + 6^2} = \sqrt{41}$

b $\overrightarrow{AB} = \begin{pmatrix} b_1 - a_1 \\ b_2 - a_2 \\ b_3 - a_3 \end{pmatrix}$ $\left|\overrightarrow{AB}\right| = \sqrt{(b_1 - a_1)^2 + (b_2 - a_2)^2 + (b_3 - a_3)^2}$

14

D

$AC \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$ $GM \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix}$

$\cos(\ ) = \dfrac{-2}{\sqrt{2} \cdot \sqrt{5}} \approx -0{,}63\ldots$ AC

GM $\cos^{-1}(0{,}63\ldots) = 51°$

$AB \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ $HM \begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix}$

$\cos(\ ) = \dfrac{2}{1 \cdot 3} \approx 0{,}66\ldots$ AB HM

$\cos^{-1}(0{,}66\ldots) = 48°$

$AF \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$ $DE \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$

$\cos(\ ) = \dfrac{1}{\sqrt{2} \cdot \sqrt{2}}$ AF DE

60°

15

a $\overrightarrow{AC} = \begin{pmatrix} -3 \\ -3 \\ 0 \end{pmatrix}$ $\overrightarrow{OF} = \begin{pmatrix} 3 \\ 3 \\ 3 \end{pmatrix}$ $\overrightarrow{AC} \cdot \overrightarrow{OF} = 0$

b $\overrightarrow{OP} = \begin{pmatrix} 3 \\ 3 \\ z \end{pmatrix}$ $\overrightarrow{EC} = \begin{pmatrix} -3 \\ 3 \\ -3 \end{pmatrix}$ $\overrightarrow{OP} \cdot \overrightarrow{EC} = 0 \Leftrightarrow 3 \cdot -3 + 3 \cdot 3 + z \cdot -3 = 0$ $z = 0$

c $\begin{pmatrix} -3 \\ 3 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} -3 \\ -3 \\ z-3 \end{pmatrix} = 0$ $z = 3$

d $\begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}$ $\begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix}$

16

a P AD Q BE

C P $(6,0,8)$ Q $(0,6,8)$

OAB 8 $\frac{1}{2} \cdot 6 \cdot 6 \cdot 8 = 144$

$DEQP$ C

$DEQP$ $3 \cdot 6\sqrt{2} = 18\sqrt{2}$

C $3\sqrt{2}$ $\frac{1}{3} \cdot 3\sqrt{2} \cdot 18\sqrt{2} =$

36

108

b DE $(x, y, z) = (6 - 3t, 3t, 4 + t)$ $X = (6 - 3t, 3t, 4 + t)$

t

$$\overrightarrow{DE} \cdot \overrightarrow{CX} = 0 \Leftrightarrow \begin{pmatrix} -3 \\ 3 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 6-3t \\ 3t \\ -4+t \end{pmatrix} = 0 \Leftrightarrow t = \frac{22}{19} \qquad X = \left(\frac{48}{19}, \frac{66}{19}, \frac{98}{19}\right)$$

Parametervoorstelling en vergelijking van een vlak

17

a $p = q = 1$ $p = 1$ $q = 2$

b $p = 2$ $q = 1$

c $p = 7$ $q = -10$

18

a $K = (4,0,2)$ $(x, y, z) = (4 + p, q, 2 + p)$

b (0,2,0) (1,0,2)

c $(t, 2 - 2t, 2t) = (4 + p, q, 2 + p)$

$4 + p = 1 + \frac{1}{2}p$ $p = -6$ $t = -2$ (-2,6,-4)

19

a $(x, y, z) = (2 - p, p + q, 2 - q)$

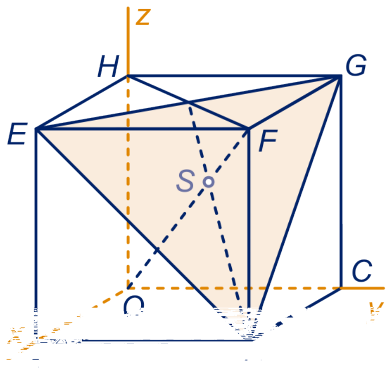
b B E G

c

d $2 - p =$

$p + q = 2 - q$ $p = q = \frac{2}{3}$

$S = \left(1\frac{1}{3}, 1\frac{1}{3}, 1\frac{1}{3}\right)$



20

a $\vec{a}$ $\vec{n}$

$\left|\vec{a}\right| \cos = 6$ $\vec{n} \cdot \vec{a} =$

$\left|\vec{n}\right| \cdot \left|\vec{a}\right| \cos = 4 \cdot 6 = 24$

$\vec{n} \cdot \vec{b} = 24$

$\vec{c}$ $\vec{n}$

$\left|\vec{c}\right| \cos = -2$ $\vec{n} \cdot \vec{c} = -8$

b P A

n

21

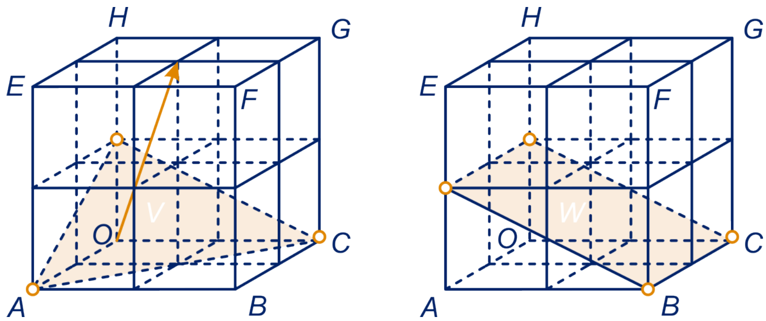
a $\vec{n} = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$ $\vec{x} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ $\vec{a} = \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix}$

$\vec{n} \cdot \vec{x} = x + y + 2z$ $\vec{n} \cdot \vec{a} = 2$

b (0,0,1) A C

c $\vec{n} = \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}$

d W B C (0,0,1) (2,0,1)



e

x

f MC M OH

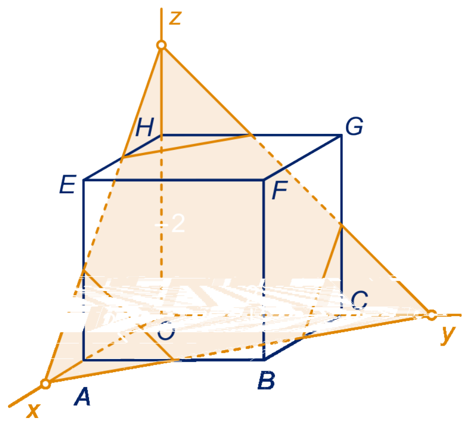
$(x, y, z) = (0,2t,1 - t)$

22

a

$(6,0,0)$ $(0,6,0)$

$(0,0,6)$



b $(4,2,0)$ $(2,4,0)$ $(4,0,2)$ $(2,0,4)$ $(0,4,2)$

$(0,2,4)$

c BG

$OAFG$

d NB OF

S

NSF

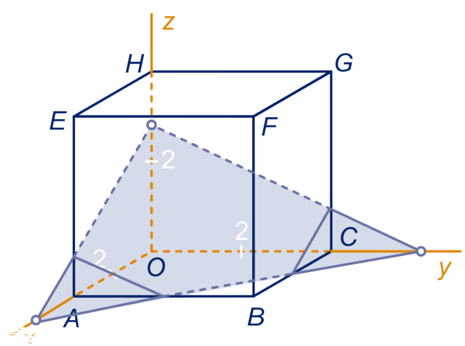
NSF BNF

23

a $(6,0,0)$ $(0,6,0)$ $(0,0,3)$

b $(4,2,0)$ $(2,4,0)$ $(0,0,3)$ $(0,4,1)$

$(4,0,1)$



c $\begin{pmatrix}1\\1\\2\end{pmatrix}$

d OEG N

EG

e $ON = 2\sqrt{6}$ $MN = 2\sqrt{3}$

$OM = 6$ $OM^2 = MN^2 + ON^2$

ON MN

OHN NFM $\frac{1}{2}\sqrt{2}$

$\angle HNO + \angle MNF = \angle HNO + \angle HON = 90^\circ$

24

a MEG $x+y+2z=d$
d $d=12$ M E G

b ON $2\sqrt{6}$

c OB T S O HT
OS HT AC AC
$OBFH$

d SOT OHT $HT \cdot OS = OT \cdot OH \Leftrightarrow$
$2\sqrt{6} \cdot OS = 2\sqrt{2} \cdot 4$ $OS = 1\frac{1}{3}\sqrt{3}$

25

a

b $\begin{pmatrix} 20 \\ 15 \\ 12 \end{pmatrix}$

c $(3{,}0{,}0)$ $(0{,}0{,}5)$

d y x
z

e $\frac{x}{3} + \frac{z}{5} = 1 \Leftrightarrow 5x + 0y + 3z = 15$ $\begin{pmatrix} 5 \\ 0 \\ 3 \end{pmatrix}$

26

a $(6{,}0{,}0)$ $(0{,}8{,}0)$ $(0{,}0{,}10)$

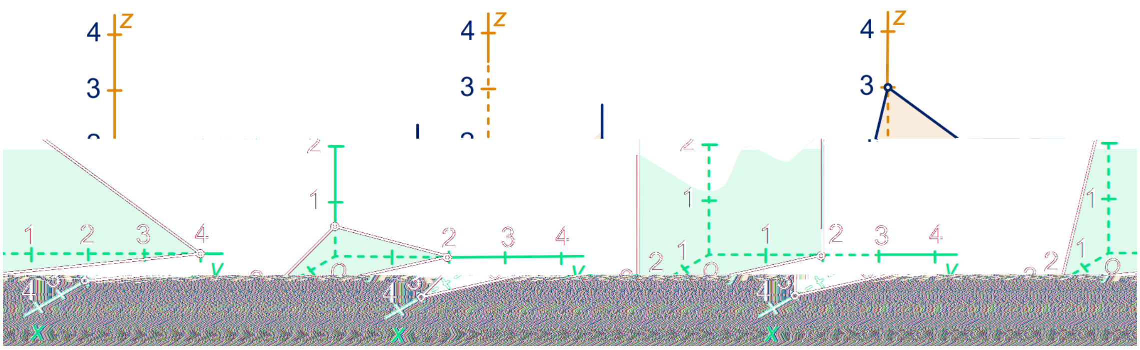
b $20x + 15y + 12z = 120$ $\begin{pmatrix} 20 \\ 15 \\ 12 \end{pmatrix}$

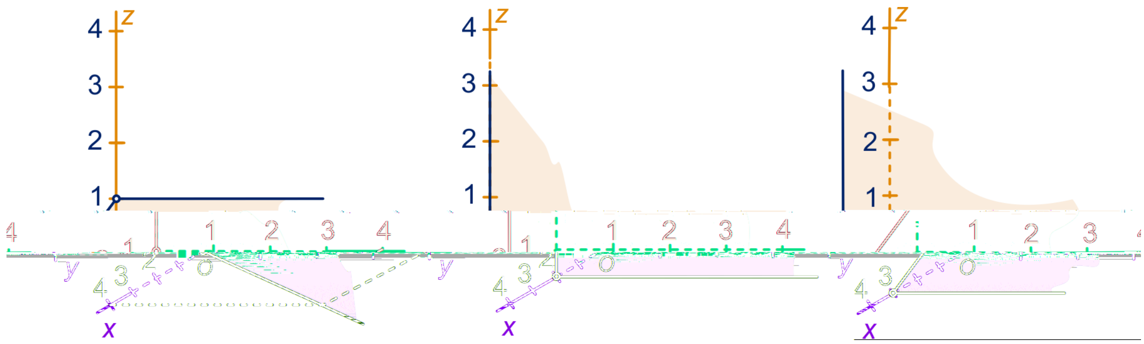
c $\frac{y}{4} + \frac{z}{5} = 1$ $\begin{pmatrix} 0 \\ 5 \\ 4 \end{pmatrix}$

d $\frac{y}{4} + \frac{z}{5} = 0$

27

$(3{,}0{,}0)$, $(0{,}2{,}0)$, $\left(0{,}0{,}\frac{1}{2}\right)$ $(3{,}0{,}0)$, $(0{,}2{,}0)$ $(2{,}0{,}0)$, $(0{,}4{,}0)$, $(0{,}0{,}3)$
$(3{,}0{,}0)$, $(0{,}0{,}1)$ $(0{,}0{,}0)$ $(2{,}0{,}0)$





28

a

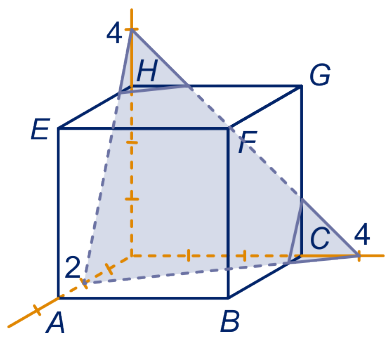
$(2,0,0)$, $(0,4,0)$, $(0,0,4)$

b $(0,t,3)$ V

$t = 1$ $(0,1,3)$

c $(2,0,0)$, $(0,3,0)$, $(0,1,1)$, $\left(\frac{1}{2},0,3\right)$, $\left(\frac{1}{2},3,0\right)$

d



29

AT $(x,y,z) = (3-t, -3+t, 2t)$

$S = (3-t, -3+t, 2t)$

t

$(3-t, -3+t, 2t)$ $y + z = 3$

$t = 2$ $S = (1,-1,4)$

30

a $8\frac{1}{2}$

b $6 = 8\frac{1}{2}$

c $a = 1\frac{1}{2}$ $b = 2$

31

a $\frac{x}{3} + \frac{y}{3} + \frac{z}{4} = 1$

b $(t,t,2t)$ $\frac{x}{3} + \frac{y}{3} + \frac{z}{4} = 1$ $t = \frac{6}{7}$

$\left(\frac{6}{7}, \frac{6}{7}, 1\frac{5}{7}\right)$

32

a $x + y + z = 3$ $x + y + z = 9$

b P $(0,0,0)$ $(9,0,0)$ $(0,9,0)$ $(0,0,9)$

$\frac{1}{3} \cdot \frac{1}{2} \cdot 9 \cdot 9 \cdot 9 = 121\frac{1}{2}$

P $\frac{1}{3}$

$\frac{23}{27} \cdot 121\frac{1}{2} = 103\frac{1}{2}$

33

$7x - y + z = 8$ $2x - y = 0$ $2x - 10y + 9z = 9$ $3y + 4z = 18$

34

$7x + 19y - 2z = 39$ $3x - 4y + z = 9$ $x - 9y + 6z = 1$ $2x - 3y + 4z = 11$

35

a $\vec{n} \cdot \overrightarrow{AP} = \begin{pmatrix} 2 \\ -3 \\ 4 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 5 \\ 3 \end{pmatrix} = 2 \cdot 0 + -3 \cdot 5 + 4 \cdot 3 = -3$ $\left|\vec{n}\right| = \sqrt{29}$

b $\frac{3}{\sqrt{29}}$

c $\vec{n} \cdot \vec{a} = 11$ A V

$\vec{n} \cdot \vec{p}$ P $2x - 3y + 4z$

36 $\frac{5}{7}\sqrt{14}$

$1\frac{1}{7}\sqrt{14}$

$x + 2y - 2z = 12$ $4\frac{1}{3}$

37 **a** $(x, y, z) = (1 + 2t, -1 + 4t, 2 + t)$

b $\frac{|1 + 2t + 2(-1 + 4t) - 2(2 + t) - 12|}{3} = 3 \Leftrightarrow t = 1$ $t = 3\frac{1}{4}$

$(3,3,3)$ $\left(7\frac{1}{2}, 12, 5\frac{1}{4}\right)$

38 $x + y + z = 9$ $(9,0,0)$

$x + y + z = 3$ $\frac{|9 + 0 + 0 - 3|}{\sqrt{3}} = 2\sqrt{3}$

39 **a** $\frac{x}{3} + \frac{y}{2} + \frac{z}{4} = 1 \Leftrightarrow 4x + 6y + 3z = 12$

b $\frac{12}{61}\sqrt{61}$

c OAC $\frac{1}{2} \cdot 2 \cdot 3$

4

d $\frac{1}{3} \cdot$ $ACH \cdot$ O $ACH =$

$ACH = \sqrt{61}$

40 **a** $\frac{24}{61}\sqrt{61}$

b 8

c 24 $\frac{1}{3} \cdot \frac{1}{2} \cdot 2 \cdot 3 \cdot 4 = 4$

$24 - 4 \cdot 4 = 8$

Hoeken

41 **a**

b

c $\tan = \frac{HD}{BD} = \frac{1}{2}\sqrt{2}$ $= 35°$

42 **a**

b

43 **a**

b

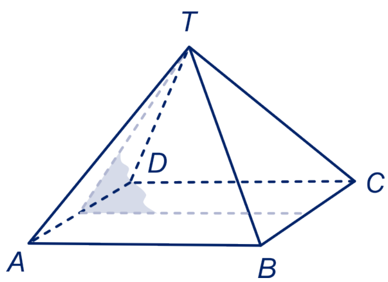
44 $90°$

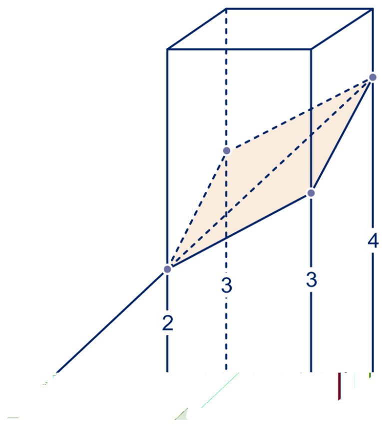
45

AD 55°

46

a





b 35°

47

a

b 0° 90°

48

a

b 35°

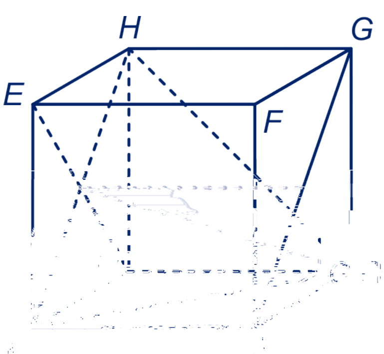
49

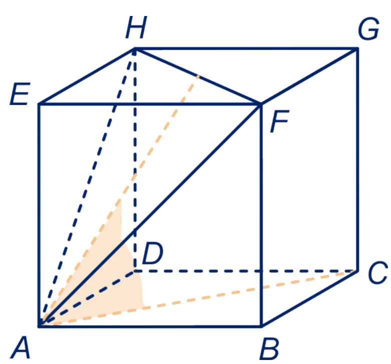
a P

Q $\tan(\ ) = \dfrac{PQ}{AP} = \sqrt{2}$

55°

b P HF Q MN PQG 55°





50

a $ABCD$ P

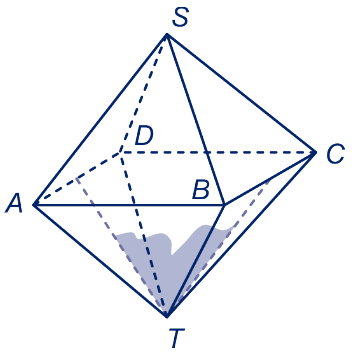
BC Q $2 \cdot \tan^{-1}\left(\frac{PQ}{PT}\right) = 2 \cdot \tan^{-1}\left(\frac{1}{\sqrt{2}}\right)$

$71°$

b DT M

$ABCD$ P $2 \cdot \tan^{-1}\left(\frac{PA}{PM}\right) = 2 \cdot \tan^{-1}\left(\sqrt{2}\right)$

$109°$



51

a A B C D

2 AB M

CMD CMD $MC = MD = \sqrt{3}$

$CD = 2$ $\sin\left(\frac{1}{2}\ \right) = \frac{1}{\sqrt{3}}$ $= 71°$

b $180°$

52

a EBP HBE

B $\frac{BP}{BE} = \frac{BE}{BH}$

$BP = 4\sqrt{3}$ $BP : HP = 2 : 1$

b EG M EMP

$\sqrt{6}$ $2\sqrt{6}$ $3\sqrt{2}$ EMP $30-60-90$

$120°$

53

a $\frac{1}{2} \cdot \frac{1}{3} \cdot 8\sqrt{2} \cdot 8\sqrt{2} \cdot 8\sqrt{2} = 170\frac{2}{3}\sqrt{2}$

b $8 \cdot 8\sqrt{3} = 64\sqrt{3}$

h $\frac{1}{3}h \cdot 64\sqrt{3} = \frac{512}{3}\sqrt{2}$ $h = \frac{8}{3}\sqrt{6}$

c T

M P

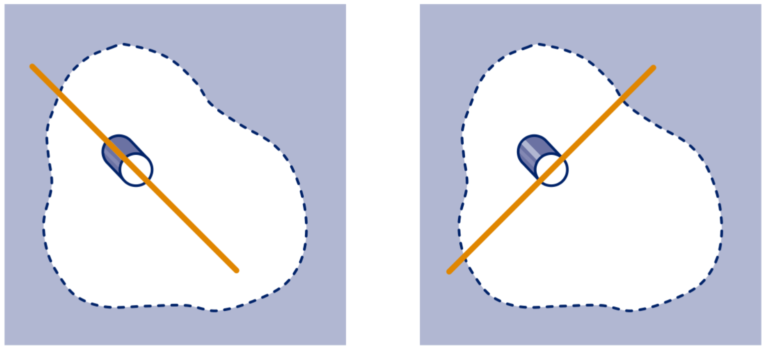
TMP $\sin \quad = \frac{h}{8}$

$=\sin^{-1}\left(\frac{1}{3}\sqrt{6}\right) = 55°$

54

a

b



c 4° 86°

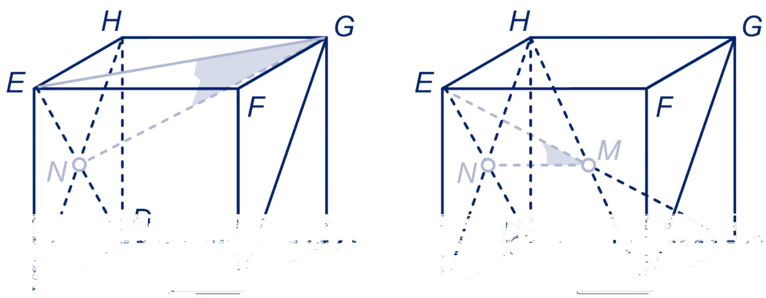
55

56

a NGE 30° DEG

b EMN $\tan(\ ) = \frac{EN}{MN} = \sqrt{2}$

$\approx 55°$



57

a $x + y + z = 6$

b $\begin{pmatrix}1\\1\\1\end{pmatrix}$ MN $N = (3+t, 6+t, 6+t)$

V $t = -3$ $N = (0,3,3)$

c $\sin(\ ) = \frac{MN}{AM} = \frac{3\sqrt{3}}{9} = \frac{1}{3}\sqrt{3}$ $= \sin^{-1}\left(\frac{1}{3}\sqrt{3}\right) = 35°$

58

a $ABHG$ $\begin{pmatrix}1\\0\\1\end{pmatrix}$

ACH $\vec{f} = \begin{pmatrix}1\\1\\1\end{pmatrix}$ $\cos(\ ) =$

$$\frac{\vec{e}\cdot\vec{f}}{\sqrt{2}\cdot\sqrt{3}}=\tfrac{1}{3}\sqrt{6}$$ $= 35°$

b $\vec{f}=\begin{pmatrix}1\\1\\1\end{pmatrix}$ $\vec{x}=\begin{pmatrix}1\\0\\2\end{pmatrix}$

$$\cos(\ )=\frac{\vec{x}\cdot\vec{f}}{\sqrt{5}\cdot\sqrt{3}}=\tfrac{1}{5}\sqrt{15}$$ $= 39°$

59 **a**

$$\cos(\ )=\frac{\overrightarrow{BA}\cdot\overrightarrow{BC}}{\left|\overrightarrow{BA}\right|\cdot\left|\overrightarrow{BC}\right|}=-\tfrac{1}{10}\sqrt{10}$$ $= 108°$

$$\cos(\ )=\frac{\overrightarrow{TB}\cdot\overrightarrow{TC}}{\left|\overrightarrow{TB}\right|\cdot\left|\overrightarrow{TC}\right|}=\frac{31}{\sqrt{43}\cdot\sqrt{29}}$$ $= 29°$

$$\cos(\ )=\frac{\left|\overrightarrow{TB}\cdot\overrightarrow{OC}\right|}{\left|\overrightarrow{TB}\right|\cdot\left|\overrightarrow{OC}\right|}=\frac{3}{\sqrt{43}}$$ $= 63°$

b $\overrightarrow{BT}$ $\overrightarrow{OT}$ 40,3°

c TAB y

(6,0,0) (0,0,10) TAB $\frac{x}{6}+\frac{z}{10}=1$

$\vec{p}=\begin{pmatrix}5\\0\\3\end{pmatrix}$

TBC $\frac{x}{-12}+\frac{y}{4}+\frac{z}{10}=1$

$\vec{q}=\begin{pmatrix}-5\\15\\6\end{pmatrix}$ $\left|\vec{p}\right|\cdot\left|\vec{q}\right|\cdot\cos(\delta)=$

$\left|\vec{p}\cdot\vec{q}\right|$ $= 86°$

60 AFH $\frac{x}{7}+\frac{y}{-7}+\frac{z}{4}=1$ $\vec{n}=\begin{pmatrix}4\\-4\\7\end{pmatrix}$

EC $\vec{p}=\begin{pmatrix}7\\-7\\4\end{pmatrix}$

EC $\left|\vec{n}\cdot\vec{p}\right|=\left|\vec{n}\right|\cdot\left|\vec{p}\right|\cdot\cos(\ )$

$= 29°$ 61°

61 **a** $2y-3z=0$ $3x-4y=0$

b BC $\begin{pmatrix} 4 \\ 3 \\ -4 \end{pmatrix}$ OAB

OAB $\begin{pmatrix} 4 \\ 3 \\ -4 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 2 \\ -3 \end{pmatrix} = \left|\begin{pmatrix} 4 \\ 3 \\ -4 \end{pmatrix}\right| \cdot \left|\begin{pmatrix} 0 \\ 2 \\ -3 \end{pmatrix}\right| \sin(\ )$ $\sin(\ ) =$

$\frac{18}{\sqrt{41} \cdot \sqrt{13}}$ $51°$

c $\begin{pmatrix} 0 \\ 2 \\ -3 \end{pmatrix}$ $\begin{pmatrix} -3 \\ 4 \\ 0 \end{pmatrix}$

$\begin{pmatrix} -3 \\ 4 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 2 \\ -3 \end{pmatrix} = \left|\begin{pmatrix} -3 \\ 4 \\ 0 \end{pmatrix}\right| \cdot \left|\begin{pmatrix} 0 \\ 2 \\ -3 \end{pmatrix}\right| \cos(\ )$ $\cos(\ ) = \frac{8}{5\sqrt{13}}$

$64°$

62 **a** $\frac{18}{13}\sqrt{13}$

b $(\ ) = \frac{\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} 4 \\ 3 \\ 2 \end{pmatrix}}{\sqrt{29}} = \frac{4}{\sqrt{29}}$ $\sin^2(\ ) + \cos^2(\ ) = 1$ $\sin(\ ) =$

$\sqrt{\frac{13}{29}}$

c $OAB = \frac{1}{2} \cdot OA \cdot OB \cdot \sin(\ ) = \frac{1}{2} \cdot 6 \cdot \sqrt{29} \cdot \sqrt{\frac{13}{29}} = 3\sqrt{13}$

d $OABC = \frac{1}{3} \cdot 3\sqrt{13} \cdot \frac{18}{13}\sqrt{13} = 18$

Het uitproduct

63 $\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$ $\begin{pmatrix} 5 \\ 2 \\ -3 \end{pmatrix}$ $\begin{pmatrix} 2 \\ 4 \\ 2 \end{pmatrix}$ $\begin{pmatrix} -5 \\ -2 \\ 3 \end{pmatrix}$ $\begin{pmatrix} -1 \\ -10 \\ 7 \end{pmatrix}$

64 **a** $\begin{pmatrix} 10 \\ 4 \\ -6 \end{pmatrix}$

b $\vec{a} \times \vec{c} = \begin{pmatrix} 5 \\ 2 \\ -3 \end{pmatrix}$ $\vec{b} \times \vec{c} = \begin{pmatrix} 2 \\ 4 \\ 2 \end{pmatrix}$ $\left(\vec{a} + \vec{b}\right) \times \vec{c} = \begin{pmatrix} 3 \\ 14 \\ -5 \end{pmatrix}$

$\left(\vec{a} + \vec{b}\right) \times \vec{c} = \vec{a} \times \vec{c} + \vec{b} \times \vec{c}$

c 0

65 **a** $\overrightarrow{AB}$ $\overrightarrow{AC}$ ABC A B C

$\overrightarrow{AB} \times \overrightarrow{AC}$ ABC

b $\overrightarrow{AB} \times \overrightarrow{AC} = \begin{pmatrix}1\\1\\2\end{pmatrix} \times \begin{pmatrix}2\\-1\\3\end{pmatrix} = \begin{pmatrix}5\\1\\-3\end{pmatrix}$

c $5x + y - 3z = c$ c

A $5x + y - 3z = -2$

66

a $\overrightarrow{OM} \times \overrightarrow{ON} = \begin{pmatrix}2\\1\\2\end{pmatrix} \times \begin{pmatrix}1\\2\\1\end{pmatrix} = \begin{pmatrix}-3\\0\\3\end{pmatrix}$ $\left|\vec{n}\right| = 3\sqrt{2}$

b $\cos(\ ) = \dfrac{\overrightarrow{OM} \cdot \overrightarrow{ON}}{\left|\overrightarrow{OM}\right| \cdot \left|\overrightarrow{ON}\right|} = \dfrac{6}{3 \cdot \sqrt{6}} = \frac{1}{3}\sqrt{6}$ $\sin(\ )$

$\sin^2(\ ) + \cos^2(\ ) = 1$ $\sin(\ ) = \frac{1}{3}\sqrt{3}$

c $\left|\overrightarrow{OM}\right| \cdot \left|\overrightarrow{ON}\right| \cdot \sin(\ ) = 3\sqrt{2}$

67

a $\overrightarrow{PQ} \times \overrightarrow{PR} = \begin{pmatrix}-6\\0\\3\end{pmatrix} \times \begin{pmatrix}0\\-4\\3\end{pmatrix} = \begin{pmatrix}12\\18\\24\end{pmatrix}$

b $\left|\begin{pmatrix}12\\18\\24\end{pmatrix}\right| = 6\sqrt{29}$

c $\cos(\ ) = \dfrac{\overrightarrow{PQ} \cdot \overrightarrow{PR}}{\left|\overrightarrow{PQ}\right| \cdot \left|\overrightarrow{PR}\right|}$ $\cos(\ ) = \frac{3}{25}\sqrt{5}$

$\sin^2(\ ) + \cos^2(\ ) = 1$ $\sin(\ ) = \frac{2}{25}\sqrt{145}$

d $\left|\overrightarrow{PQ}\right| \cdot \left|\overrightarrow{PR}\right| \cdot \sin(\ ) = 15\sqrt{5} \cdot \frac{2}{25}\sqrt{145} = 6\sqrt{29}$

e PQR $\overrightarrow{PQ} \times \overrightarrow{PR}$ $\begin{pmatrix}2\\3\\4\end{pmatrix}$

PQR $2x + 3y + 4z = 24$

(12,0,0) (0,8,0) (0,0,6)

68

a $\vec{r}$ $\vec{p}$ $\vec{q}$

b 120

69

a $B(7,3,0)$ $E(5,-2,6)$ $F(7,1,6)$ $G(2,1,6)$

b 15 90

c 90

70

a $B(9,5,0)$ $E(0,-3,5)$ $F(3,2,5)$ $G(0,2,5)$

b $\left(\overrightarrow{OA} \times \overrightarrow{OC}\right) \cdot \overrightarrow{OH} = \begin{pmatrix}0\\0\\15\end{pmatrix} \cdot \begin{pmatrix}-3\\-3\\5\end{pmatrix} = 75$ 75

c $\frac{1}{2} \cdot 75 = 37\frac{1}{2}$

d ABC

$\frac{1}{3} \cdot 37\frac{1}{2} = 12\frac{1}{2}$

71

a D A A C
y H z $\frac{1}{6}$
A C D H

$$\frac{1}{6}\left(\begin{pmatrix}1\\1\\0\end{pmatrix} \times \begin{pmatrix}0\\1\\1\end{pmatrix}\right) \cdot \begin{pmatrix}1\\0\\1\end{pmatrix} = \frac{1}{6}\left(\begin{pmatrix}1\\-1\\1\end{pmatrix}\right) \cdot \begin{pmatrix}1\\0\\1\end{pmatrix} = \frac{1}{3}$$

b $\frac{1}{3} \cdot \frac{1}{2} = \frac{1}{6}$
$1 - 4 \cdot \frac{1}{6} = \frac{1}{3}$

72

$\frac{1}{6}$

$$\left(\overrightarrow{MP} \times \overrightarrow{MN}\right) \cdot \overrightarrow{MQ}$$

$$\overrightarrow{MP} = \begin{pmatrix}1\\2\\0\end{pmatrix} \quad \overrightarrow{MN} = \begin{pmatrix}1\\1\\-2\end{pmatrix} \quad \overrightarrow{MQ} = \begin{pmatrix}-1\\2\\-2\end{pmatrix}$$

$$\left(\overrightarrow{MP} \times \overrightarrow{MN}\right) \cdot \overrightarrow{MQ} = \begin{pmatrix}-4\\2\\1\end{pmatrix} \cdot \begin{pmatrix}-1\\2\\-2\end{pmatrix} = 6$$ 1

73

$\frac{1}{6}$ $\left(\overrightarrow{AH} \times \overrightarrow{AM}\right) \cdot \overrightarrow{AC}$

$$\left(\overrightarrow{AH} \times \overrightarrow{AM}\right) \cdot \overrightarrow{AC} = \begin{pmatrix}-8\\4\\-4\end{pmatrix} \cdot \begin{pmatrix}-3\\2\\0\end{pmatrix} = 32$$ $\frac{1}{6} \cdot 32 = 5\frac{1}{3}$

Determinant en uitproduct

74

a 11 17
b -11 -17 11 17

75

a
b $\vec{a}$ $\vec{b}$
90° – – 90°
$$\left|\vec{a}\right| \cdot \left|\vec{b}\right| \sin(\ ) = \left|\vec{a}\right| \cdot \left|\vec{b}_R\right| \sin(\ )$$ $\vec{b}$ $\vec{b}_R$
$\sin(\ ) = \cos(\ )$
$\sin(\ ) = -\cos(\ )$

c

76

a $\begin{vmatrix}-1 & 3\\3 & 2\end{vmatrix} = -11$ 11 $\begin{vmatrix}-1 & 5\\3 & 2\end{vmatrix} =$
-17 17

b $\overrightarrow{AB} = \begin{pmatrix} 6 \\ -4 \end{pmatrix}$ $\overrightarrow{AC} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$ $\begin{vmatrix} 6 & 4 \\ -4 & 3 \end{vmatrix} = 34$

$ABC = 17$

c

77

a 10 14 -14 0

b $-2 \cdot 10 = -20$

c 24

d -20

78

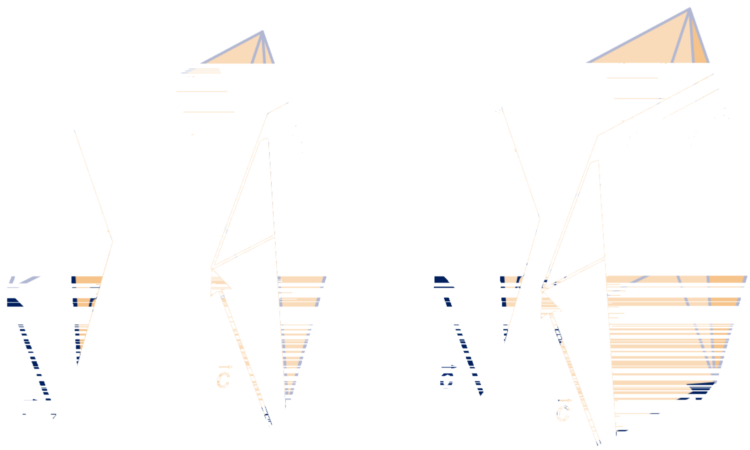
a

$\frac{1}{2}$

b

79

a

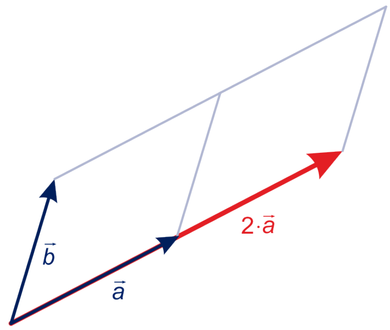


$\left(\vec{a}, \vec{b}\right) +$ $\left(\vec{a}, \vec{c}\right)$

$\left(\vec{a}, \vec{b} + \vec{c}\right)$

b

$2 \cdot \vec{a}$ $\vec{b}$



80 **a** $\left(\vec{a}, \vec{b}\right) =$ $\left(a_1 \cdot \vec{e}_1 + a_2 \cdot \vec{e}_2, \vec{b}\right) =$

$\left(a_1 \cdot \vec{e}_1, \vec{b}\right) +$ $\left(a_2 \cdot \vec{e}_2, \vec{b}\right) =$

$a_1 \cdot$ $\left(\vec{e}_1, \vec{b}\right) + a_2 \cdot$ $\left(\vec{e}_2, \vec{b}\right) =$

b $\left(\vec{e}_1, \vec{e}_2\right)$

1 1 $\left(\vec{e}_1, \vec{e}_2\right) =$

- $\left(\vec{e}_2, \vec{e}_1\right)$ $\left(\vec{e}_1, \vec{e}_1\right)$ $\left(\vec{e}_2, \vec{e}_2\right)$

81 $C = (t, 2t + 7)$ $\overrightarrow{AB} = \begin{pmatrix} 8 \\ -2 \end{pmatrix}$ $\overrightarrow{AC} = \begin{pmatrix} t+3 \\ 2t \end{pmatrix}$ $\left(\overrightarrow{AB}, \overrightarrow{AC}\right) = \pm 30$

$\Leftrightarrow 18t - 6 = \pm 30$ $C = (5,10)$ $C = \left(1\frac{1}{2}, -3\right)$

82 $\left(\vec{a}, \vec{b}, \vec{c}\right)$ $\left(\vec{b}, \vec{c}, \vec{a}\right)$ $\left(\vec{c}, \vec{a}, \vec{b}\right)$

83 **a**

b $\left(\vec{e}_1, \vec{e}_1, \vec{e}_2\right) = 0$ $\left(\vec{e}_1, \vec{e}_1, \vec{e}_2\right)$

$\left(\vec{e}_1, \vec{e}_2, \vec{e}_3\right) =$ $\left(\vec{e}_2, \vec{e}_3, \vec{e}_1\right) =$ $\left(\vec{e}_3, \vec{e}_1, \vec{e}_2\right) = 1$ $\left(\vec{e}_1, \vec{e}_3, \vec{e}_2\right) =$

$\left(\vec{e}_2, \vec{e}_1, \vec{e}_3\right) =$ $\left(\vec{e}_3, \vec{e}_2, \vec{e}_1\right) = -1$

1

c

84 2 -14 -51

85 **a** $\overrightarrow{AB} = \begin{pmatrix} 2 \\ -2 \\ -1 \end{pmatrix}$ $\overrightarrow{BC} = \begin{pmatrix} 1 \\ -3 \\ -1 \end{pmatrix}$ $\overrightarrow{AE} = \begin{pmatrix} 0 \\ 0 \\ 10 \end{pmatrix}$

A H $\overrightarrow{BC} + \overrightarrow{AE}$ $H = (3,-3,9)$

b $\left(\overrightarrow{AB}, \overrightarrow{BC}, \overrightarrow{AE}\right) =$

$$\begin{vmatrix} 2 & 1 & 0 \\ -2 & -3 & 0 \\ -1 & -1 & 10 \end{vmatrix}$$

c $\frac{1}{6} \cdot 40 = 6\frac{2}{3}$ $6\frac{2}{3}$

86 $\frac{1}{6}$

$\frac{1}{6}$

$\frac{1}{6}$

87 **a** $\left(\vec{a}, \vec{r}, \vec{s}\right)$ $\vec{a} = \begin{pmatrix} a \\ b \\ 2 \end{pmatrix}$

b $a + b = 4$

c $\begin{vmatrix} a-3 & 2 & 1 \\ b & 0 & 1 \\ 2 & 1 & 1 \end{vmatrix} = 0$ $a + b = 7$

88 **a** $x + 4y - 3z = 0$

b

89

90 $\begin{pmatrix} 5 \\ 2 \\ -3 \end{pmatrix}$ $\begin{pmatrix} 2 \\ 4 \\ 2 \end{pmatrix}$ $\begin{pmatrix} -1 \\ -10 \\ 7 \end{pmatrix}$

91 $2 \cdot (|\overrightarrow{AB} \times \overrightarrow{BC}| + |\overrightarrow{AB} \times \overrightarrow{AE}| + |\overrightarrow{BC} \times \overrightarrow{AE}|) =$

$= 2 \cdot \left|\begin{pmatrix} -1 \\ -1 \\ -4 \end{pmatrix}\right| + 2 \cdot \left|\begin{pmatrix} -20 \\ -20 \\ 0 \end{pmatrix}\right| + 2 \cdot \left|\begin{pmatrix} -30 \\ -10 \\ 0 \end{pmatrix}\right| = 46\sqrt{2} + 20\sqrt{10}$

92 $\vec{a} \times \vec{b}$ V $\vec{c} \times \vec{d}$ W

$\vec{r} = \left(\vec{a} \times \vec{b}\right) \times \left(\vec{c} \times \vec{d}\right)$ V W

V W

Extra opgaven

1 $\begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix}$ $\begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix}$ $\cos(\ ) = -\frac{1}{5}$ $= 102°$

2 **a**

LR x R_s LQ

y Q_s

PQ_sR_s

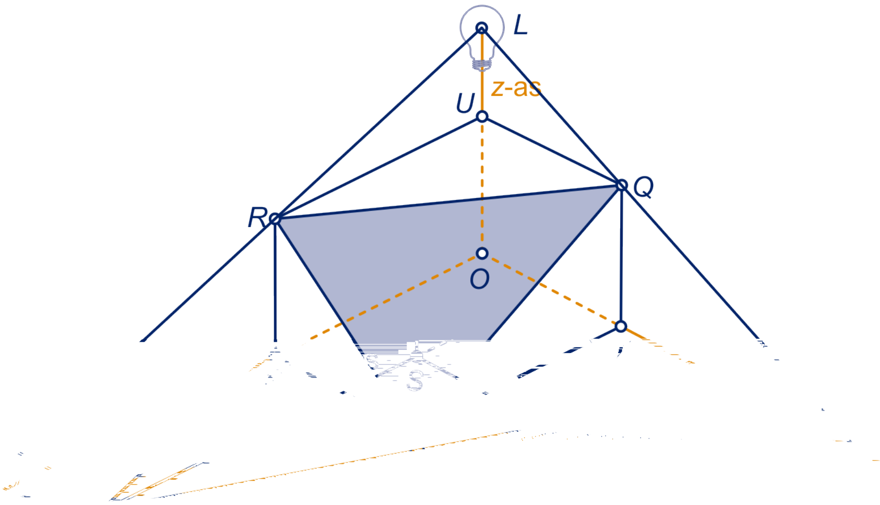
b $SR_s = 3 \cdot RS = 9$ $TQ_s = 2 \cdot TQ = 6$

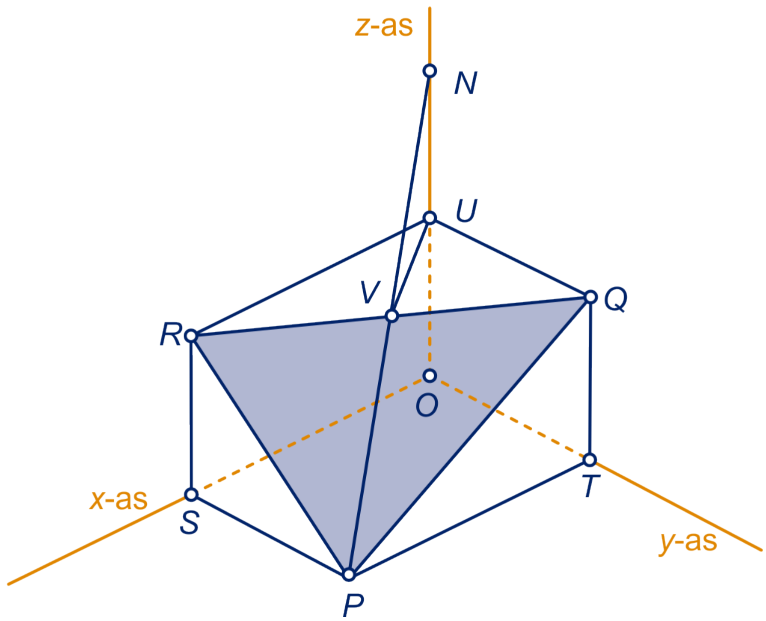
$PSR_s = 18$ $PTQ_s = 18$

$OSPT = 24$ $OR_sQ_s = 75$

15

c





U OP RQ

V PV z N

d $\overrightarrow{PQ}=\begin{pmatrix}-6\\0\\3\end{pmatrix}$ $\overrightarrow{PR}=\begin{pmatrix}0\\-4\\3\end{pmatrix}$

$\overrightarrow{PQ}\times\overrightarrow{PR}=\begin{pmatrix}12\\18\\24\end{pmatrix}$

e $\frac{1}{2}\left|\begin{pmatrix}12\\18\\24\end{pmatrix}\right| = 3\sqrt{29}$

f PQR $\begin{pmatrix}2\\3\\4\end{pmatrix}$

$2x + 3y + 4z = 24$ z $(0,0,6)$

3

a $\overrightarrow{AH}=\begin{pmatrix}-3\\0\\4\end{pmatrix}$ $\overrightarrow{CH}=\begin{pmatrix}0\\-2\\4\end{pmatrix}$ $\vec{u}=\begin{pmatrix}8\\12\\6\end{pmatrix}$

b ACH $\frac{1}{2}\cdot\left|\vec{u}\right| = \sqrt{61}$

c $\vec{u}$ $\overrightarrow{CE}=\begin{pmatrix}3\\-2\\4\end{pmatrix}$ $90° -$

$\cos(90° - \ ) = \frac{12}{\sqrt{61}\cdot\sqrt{29}}$ $= 17°$

d OP $(x, y, z) = (4t, 6t, 3t)$

ACH $4x + 6y + 3z = 12$ t

P $4\cdot 4t + 6\cdot 6t + 3\cdot 3t = 12$ $t = \frac{12}{61}$

$P = \left(\frac{48}{61}, \frac{72}{61}, \frac{36}{61}\right)$

4

a $OSRH$ $\frac{1}{3}\cdot\frac{1}{2}\cdot 3\cdot 3\cdot 6 = 9$

$6^4 - 4\cdot 9 = 180$

b $ABTRU$ T BG

CQ U OS AH

c TBC TGQ

2

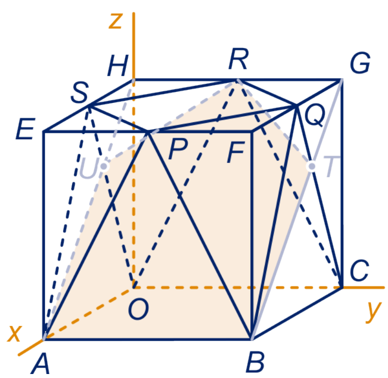
$BT = \frac{2}{3}\cdot 6\sqrt{2} = 4\sqrt{2}$

$ABTU =$

$4\sqrt{2}\cdot 6 = 24\sqrt{2}$

$RTU = \frac{1}{2}\cdot 2\sqrt{2}\cdot 6 = 6\sqrt{2}$

$24\sqrt{2} + 6\sqrt{2} = 30\sqrt{2}$



d x $(12,0,0)$ y

$(0,12,0)$

$\frac{x}{12} + \frac{y}{12} + \frac{z}{c} = 1$

c $(6,3,6)$ $c = 24$

$2x + 2y + z = 24$

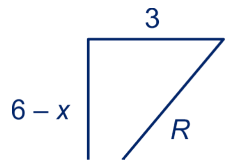
e $\begin{pmatrix}1\\0\\0\end{pmatrix}$ $\begin{pmatrix}2\\2\\1\end{pmatrix}$

$\cos(\ )$ $\cos(\ ) = \frac{2}{3}$ $= 48°$

f

M

x



3

$3\sqrt{2}$

R $(6-x)^2 +$

$3^2 = R^2$ $x^2 + \left(3\sqrt{2}\right)^2 = R^2$ $x = 2\frac{1}{4}$

M $\left(3{,}3{,}2\frac{1}{4}\right)$

g h

6 6

$\frac{1}{2}h$

$36 - \frac{1}{2}h^2$ $h = 4\sqrt{3}$

5

O $\frac{x}{3} + \frac{y}{4} + \frac{z}{5} = 1$

$20x + 15y + 12z - 60 = 0$ O

$$\frac{|20 \cdot 0 + 15 \cdot 0 + 12 \cdot 0 - 60|}{\sqrt{20^2 + 15^2 + 12^2}} = \frac{60}{\sqrt{769}}$$

6

a $\frac{x}{1} + \frac{z}{\sqrt{3}} = 1$ $\frac{y}{1} + \frac{z}{\sqrt{3}} = 1$

$\begin{pmatrix} \sqrt{3} \\ 0 \\ 1 \end{pmatrix}$ $\begin{pmatrix} 0 \\ \sqrt{3} \\ 1 \end{pmatrix}$

$$\cos(\ ) = \frac{\sqrt{3} \cdot 0 + 0 \cdot \sqrt{3} + 1 \cdot 1}{2 \cdot 2} = \frac{1}{4} \qquad = 76^\circ$$

b $\begin{pmatrix} \tan(\ ) \\ 0 \\ 1 \end{pmatrix}$

$\begin{pmatrix} 0 \\ \tan(\ ) \\ 1 \end{pmatrix}$ $\frac{1}{\tan^2(\ ) + 1} = \frac{1}{2}\sqrt{2} \Leftrightarrow \cos^2(\ ) = \frac{1}{2}\sqrt{2}$ $= 33^\circ$

7

a OA BC $(-12{,}0{,}0)$

T

$(x, y, z) = (-12 + 6t{,}0{,}5t)$

b P BT O BT

V $3x + 3y - 5z = 0$

BT $(x, y, z) = (0{,}0{,}10) + t\,(3{,}3{,}-5)$ P

t $3 \cdot 3t + 3 \cdot 3t - 5 \cdot (10 - 5t) = 0$ $t = \frac{50}{43}$

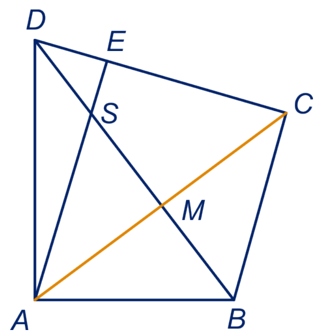
$P = \left(\frac{150}{43}, \frac{150}{43}, \frac{180}{43}\right)$

c Q BT V O BC
V $3x - y = 0$
BC $(x, y, z) = (0,4,0) + t\,(3,\text{-}1,0)$ Q
t $3 \cdot 3t - (4 - t) = 0 \Leftrightarrow t = \frac{2}{5}$ $Q = \left(1\frac{1}{5}, 3\frac{3}{5}, 0\right)$

8

a ABD BCD
$\begin{pmatrix}4\\0\\3\end{pmatrix}$ $\begin{pmatrix}0\\4\\3\end{pmatrix}$ $\cos(\ ) = \frac{9}{25}$ $= 69°$
$111°$

b
ABD BCD
AC M
AC BD
$MA \cdot BD = AD \cdot AB$
ABD
$MA = \frac{15}{34}\sqrt{34}$ $AC = \frac{15}{17}\sqrt{34}$



c AM BD M
V BD BD
BD $(x, y, z) = (0,0,4) + t\,(3,3,\text{-}4)$
V $3x + 3y - 4z = 9$
$t = \frac{25}{34}$ $\left(\frac{75}{34}, \frac{75}{34}, \frac{36}{34}\right)$

d AE AE DC
ADB $\cos(\ ) = \frac{5}{\sqrt{34}}$ $\sin(\ ) = \frac{3}{\sqrt{34}}$
$\sin(2\ ) = 2 \cdot \sin(\ ) \cdot \cos(\ ) = \frac{15}{17}$ $AE = AD \cdot \sin(2\ ) = \frac{75}{17}$

e AMC A M C
AMC $\frac{15}{\sqrt{34}}$ $\frac{15}{\sqrt{34}}$ $3\sqrt{2}$

$$2 \cdot \sin^{-1}\left(\frac{1\frac{1}{2}\sqrt{2}}{\frac{15}{\sqrt{34}}}\right) \approx 111°$$

6 Ruimte

ACP

AC

$OBFD$

B MN

AC EG

$\frac{1}{3}G \cdot h$ G

h

DE X

t

$OBFH$ AC

$(0,t,3)$ t

G

h $\frac{1}{3}G \cdot h$

N $(3+t,6+t,6+t)$

ACH

$(\ )$

$OA \cdot OB \cdot \sin(\ )$

$\frac{1}{3} \cdot$.

$ABCD.EFGH$ M

DH AM

G

h $\frac{1}{3} \cdot G \cdot h$