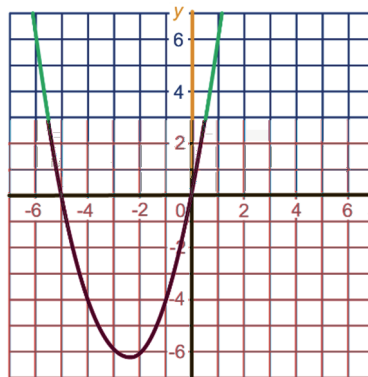
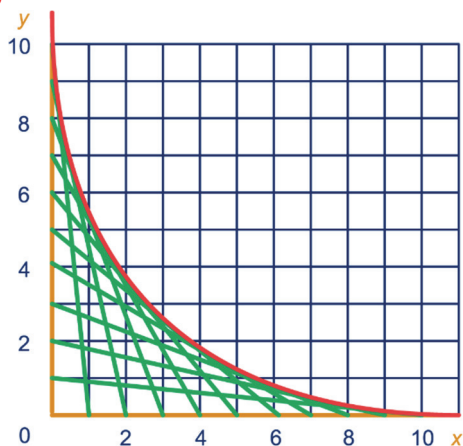


H29 PARABOLEN HAVO

29.0 INTRO

1 ab



c ...

d $x = -2\frac{1}{2}$

4 $(3, 9)$; $(-2\frac{1}{2}, -6)$

5 a

x	-3	-2	-1	0	1	2	3
$y = x^2$	9	4	1	0	1	4	9
$y = \frac{1}{10}x^2$	0,9	0,4	0,1	0	0,1	0,4	0,9
$y = \frac{1}{2}x^2$	4,5	2	0,5	0	0,5	2	4,5
$y = 2x^2$	18	8	2	0	2	8	18

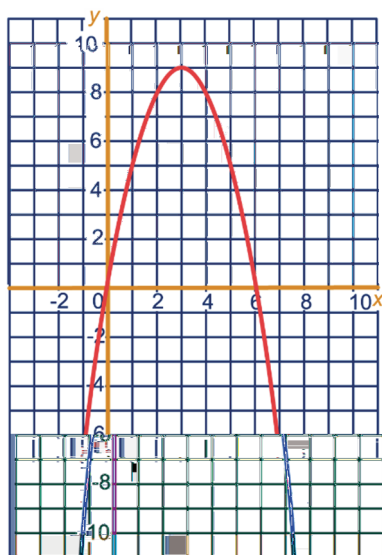
29.1 PARABOLEN

2 a $6 - x$

b $y = x(6 - x)$

c

x	-1	0	1	2	3	4	5	6	7
y	-7	0	5	8	9	8	5	0	-7



d ...

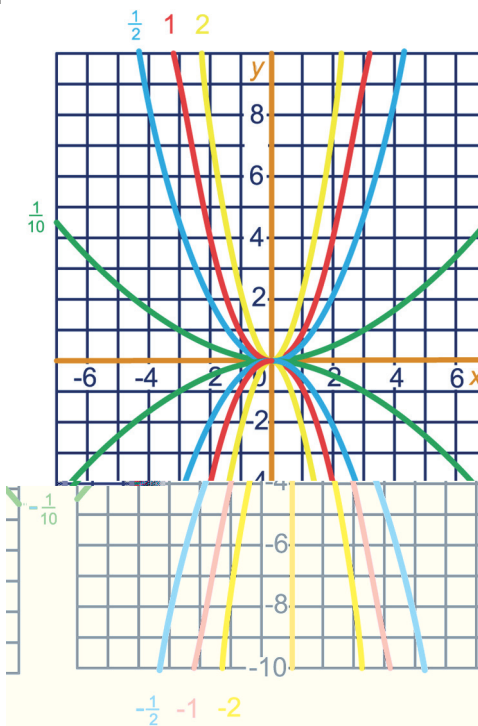
e $x = 3$

3 a $y = x(x + 5)$

b

x	-6	-5	-4	-3	-2	-1	0	1	2
y	6	0	-4	-6	-6	-4	0	6	14

bd



c

x	-3	-2	-1	0	1	2	3
$y = -x^2$	-9	-4	-1	0	-1	-4	-9
$y = -\frac{1}{10}x^2$	-0,9	-0,4	-0,1	0	-0,1	-0,4	-0,9
$y = -\frac{1}{2}x^2$	-4,5	-2	-0,5	0	-0,5	-2	-4,5
$y = -2x^2$	-18	-8	-2	0	-2	-8	-18

e Dalparabool als $c > 0$,
een bergparabool als $c < 0$.

f Ze zijn elkaars spiegelbeeld in de x-as.

g Dan is $y = 0$, dat is een rechte lijn, dat is de
vergelijking van de x-as.

- 6 $y = cx^2$
 $3 = c \cdot 1^2$ (invullen het punt (1, 3))
 $3 = c$
- $y = cx^2$
 $2 = c \cdot (-5)^2$ (invullen het punt (-5, 2))
 $2 = 25c$
 $\frac{2}{25} = c$
- $y = cx^2$
 $-3 = c \cdot 3^2$ (invullen het punt (3, -3))
 $-3 = 9c$
 $-\frac{1}{3} = c$
- 7 $y = cx^2$
 $4 = c \cdot 5^2$ (invullen het punt (5, 4) of (-5, 4))
 $4 = 25c$
 $\frac{4}{25} = c$

$$x^2 + 6x = 16$$

$$x^2 + 6x - 16 = 0$$

$$(x + 8)(x - 2) = 0$$

$$x = -8 \text{ of } x = 2$$

➤ MIN 16
 ➤ ONTBINDEN

$$x^2 + 16 = 8x$$

$$x^2 - 8x + 16 = 0$$

$$(x - 4)^2 = 0$$

$$x = 4$$

➤ MIN 8x
 ➤ ONTBINDEN

$$3(x + 1) = x^2 + 5$$

$$3x + 3 = x^2 + 5$$

$$x^2 - 3x + 2 = 0$$

$$(x - 2)(x - 1) = 0$$

$$x = 2 \text{ of } x = 1$$

➤ HAAKJES WEG
 ➤ MIN 3x, MIN 3
 ➤ ONTBINDEN

$$(x + 1)(x + 3) = 1 - x^2$$

$$x^2 + 4x + 3 = 1 - x^2$$

$$2x^2 + 4x + 2 = 0$$

$$x^2 + 2x + 1 = 0$$

$$(x + 1)(x + 1) = 0$$

$$x = -1$$

➤ HAAKJES WEG
 ➤ PLUS x^2 , MIN 1
 ➤ DELEN DOOR 2
 ➤ ONTBINDEN

29.2 HERHALING

8 $12 - 18x$
 $8x^2 - 20x$
 $-4x + 10x^2$
 $3x - 8$
 $-2x^2 - 39x - 10$
 $15x^2 - 9x - 13$
 $22 + 2x^2 - 6x$
 $-3x + 4y$
 $5x + 3y$

$$x^2 - 5x = 6$$

$$x^2 - 5x - 6 = 0$$

$$(x - 6)(x + 1) = 0$$

$$x = 6 \text{ of } x = -1$$

➤ MIN 6
 ➤ ONTBINDEN

$$3 - 4x = 1 - 2x^2$$

$$2x^2 - 4x + 2 = 0$$

$$x^2 - 2x + 1 = 0$$

$$(x - 1)(x - 1) = 0$$

$$x = 1$$

➤ PLUS $2x^2$, MIN 1
 ➤ DELEN DOOR 2
 ➤ ONTBINDEN

9 $x^2 - 10x + 21$ $3x^2 + 5x - 8$
 $6x^2 + 22x - 8$ $2x^2 + 3x - 2$
 $p^2 + 6p + 9$ $25 - 10q + q^2$
 $-p^2 + 4q^2$ $4p^2 - 12pq + 9q^2$

$$12 - 11x = x^2$$

$$x^2 + 11x - 12 = 0$$

$$(x + 12)(x - 1) = 0$$

$$x = -12 \text{ of } x = 1$$

➤ PLUS 11x, MIN 12
 ➤ ONTBINDEN

$$3x^2 = 6x - 3$$

$$3x^2 - 6x + 3 = 0$$

$$x^2 - 2x + 1 = 0$$

$$(x - 1)(x - 1) = 0$$

$$x = 1$$

➤ MIN 6x, PLUS 3
 ➤ DELEN DOOR 3
 ➤ ONTBINDEN

10 $x(x + 7)$ $(x - 3)^2$
 $x(x - 10)$ $(x + 5)^2$
 $(x - 7)(x - 1)$ $(x - 6)^2$
 $(x - 3)(x + 9)$ $(2x - 3)^2$

$$5x^2 = -15x$$

$$5x^2 + 15x = 0$$

$$x^2 - 3x = 0$$

$$x(x - 3) = 0$$

$$x = 0 \text{ of } x = 3$$

➤ PLUS 15x
 ➤ DELEN DOOR 5
 ➤ ONTBINDEN

11 $x^2 + 10x = -16$ ➤ PLUS 16
 $x^2 + 10x + 16 = 0$ ➤ ONTBINDEN
 $(x + 2)(x + 8) = 0$
 $x = -2 \text{ of } x = -8$

$10x = x^2$ ➤ MIN x^2
 $10x - x^2 = 0$ ➤ ONTBINDEN
 $x(10 - x) = 0$
 $x = 0 \text{ of } x = 10$

$$2(x^2 - 2) = 4(x^2 - 3)$$

$$2x^2 - 4 = 4x^2 - 12$$

$$0 = 2x^2 - 8$$

$$0 = x^2 - 4$$

$$0 = (x - 2)(x + 2)$$

$$x = 2 \text{ of } x = -2$$

➤ HAAKJES WEG
 ➤ MIN $2x^2$, PLUS 4
 ➤ DELEN DOOR 2
 ➤ ONTBINDEN

29.3 PARABOLEN TEKENEN

- 12 a** $h = 0,0625 \cdot 40^2 = 100$ m
b als $x = 0$, $h = 0,0625 \cdot 0^2 = 0$ m
als $x = 10$, $h = 0,0625 \cdot 10^2 = 6,25$ m
als $x = 20$, $h = 0,0625 \cdot 20^2 = 25$ m
als $x = 30$, $h = 0,0625 \cdot 30^2 = 56,25$ m
als $x = 40$, $h = 0,0625 \cdot 40^2 = 100$ m
c $x = 35$, dan $h = 0,0625 \cdot 35^2 = 76,5625$ m
De hoogte boven de Wupper is dan $100 - 76,5625 = 23,4375$ m.

- 13 a** $x^2 - 6x = 0$
 $x(x-6) = 0$
 $x = 0$ of $x = 6$
Dus de nulpunten zijn 0 en 6.

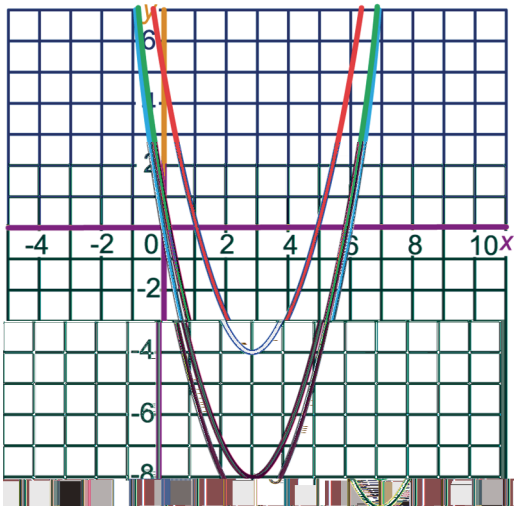
b vergelijking symmetrieas: $x = \frac{0+6}{2} = 3$

c $y = 3^2 - 6 \cdot 3 = -9$, Top(3, -9).

d

x	-1	0	$\frac{1}{2}$	1	2	3	4	5	$5\frac{1}{2}$	6	7
y	7	0	$-2\frac{3}{4}$	-5	-8	-9	-8	-5	$-2\frac{3}{4}$	0	7

ef



g 9

h $y = x^2 - 6x + 9 = (x - 3)^2$

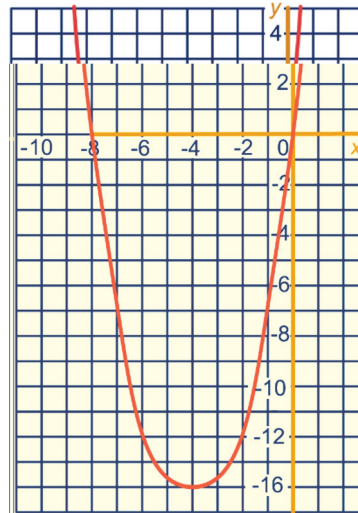
- 14 a** $x^2 + 8x = 0$
 $x(x+8) = 0$
 $x = 0$ of $x = -8$
Dus de nulpunten zijn 0 en -8.

b vergelijking symmetrieas: $x = \frac{-8+0}{2} = -4$

c $y = (-4)^2 + 8 \cdot (-4) = -16$
Top(-4, -16).

d

x	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1
y	9	0	-7	-12	-15	-16	-15	-12	-7	0	9



e 16

f $y = x^2 + 8x + 16 = (x + 4)^2$

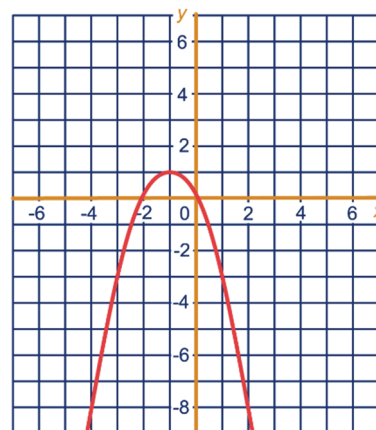
- 15 a** $-x^2 - 2x = 0$
 $-x(x+2) = 0$
 $x = 0$ of $x = -2$
Dus de nulpunten zijn 0 en -2.

b vergelijking symmetrieas: $x = \frac{-2+0}{2} = -1$

c $y = -(-1)^2 - 2 \cdot (-1) = -1 + 2 = 1$
Top(-1, 1).

d

x	-6	-5	-4	-3	-2	-1	0	1	2	3	4
y	-24	-15	-8	-3	0	1	0	-3	-8	-15	-24



e -1

- 16 a** snijpunt y-as $\Rightarrow x = 0$
 $y = 0^2 - 2 \cdot 0 + 4 = 4$
(0,4)

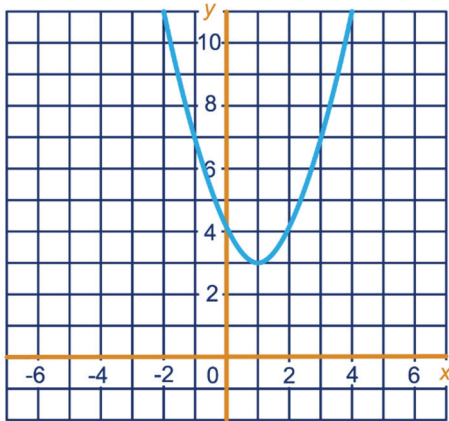
b $x^2 - 2x + 4 = 4$
 $x^2 - 2x = 0$
 $x(x-2) = 0$
 $x = 0$ of $x = 2$

c vergelijking symmetrieas: $x = \frac{0+2}{2} = 1$

d $y = 1^2 - 2 \cdot 1 + 4 = 3$
Top(1, 3).

e

x	-2	-1	0	1	2	3	4
y	12	7	4	3	4	7	12



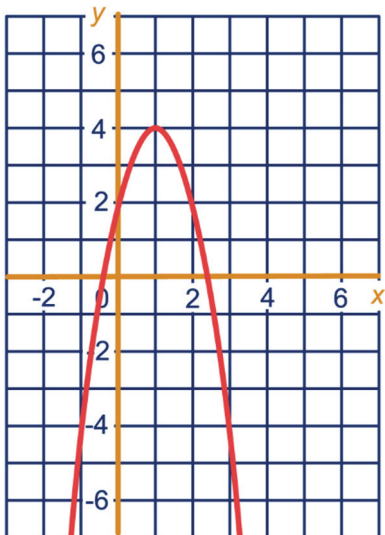
17 a $y = -2 \cdot 0^2 + 4 \cdot 0 + 2 = 2$
Snijpunt y-as (0, 2).

b $-2x^2 + 4x + 2 = 2$
 $-2x^2 + 4x = 0$
 $x^2 - 2x = 0$
 $x(x - 2) = 0$
 $x = 0$ of $x = 2$

c vergelijking symmetrieas: $x = \frac{0+2}{2} = 1$

d $y = -2 \cdot 1^2 + 4 \cdot 1 + 2 = 4$
Top(1, 4).

e



18

$$y = x^2 - 2x$$

Nulpunten:

$$x^2 - 2x = 0$$

$$x(x - 2) = 0$$

$$x = 0 \text{ of } x = 2$$

Snijpunt y-as:

$$y = 0^2 + 2 \cdot 0 = 0$$

(0, 0)

Symmetrieas:

$$x = \frac{0+2}{2} = 1$$

$$y = 1^2 - 2 \cdot 1 = -1$$

Top(1, -1).

x	-2	-1	0	1	2	3	4
y	8	3	0	-1	0	3	8

$$y = -x^2 + 5x - 2$$

Snijpunt y-as:

$$y = -0^2 + 5 \cdot 0 - 2 = -2$$

(0, -2)

$$-x^2 + 5x - 2 = -2$$

$$-x^2 + 5x = 0$$

$$-x(x - 5) = 0$$

$$x = 0 \text{ of } x = 5$$

Symmetrieas:

$$x = \frac{0+5}{2} = 2\frac{1}{2}$$

$$y = -(2\frac{1}{2})^2 + 5 \cdot 2\frac{1}{2} - 2 = 4\frac{1}{4}$$

Top(2 $\frac{1}{2}$, 4 $\frac{1}{4}$).

x	-1	0	1	2	2 $\frac{1}{2}$	3	4	5	6
y	-8	-2	2	4	4 $\frac{1}{4}$	4	2	-2	-8

$$y = (x + 2)(x - 8)$$

Nulpunten:

$$(x + 2)(x - 8) = 0$$

$$x = -2 \text{ of } x = 8$$

Snijpunt y-as:

$$y = (0 + 2)(0 - 8) = -16$$

(0, -16)

Symmetrieas:

$$x = \frac{-2+8}{2} = 3$$

$$y = (3 + 2)(3 - 8) = -25$$

Top(3, -25).

x	0	1	2	3	4	5	6
y	-16	-21	-24	-25	-24	-21	-16

$$y = 2x^2 - 3x + 1$$

Snijpunt y - as:

$$y = 2 \cdot 0^2 - 3 \cdot 0 + 1 = 1$$

(0, 1)

$$2x^2 - 3x + 1 = 1$$

$$2x^2 - 3x = 0$$

$$2x(x - 1\frac{1}{2}) = 0$$

$$x = 0 \text{ of } x = 1\frac{1}{2}$$

Symmetrieas:

$$x = \frac{0 + 1\frac{1}{2}}{2} = \frac{3}{4}$$

$$y = 2 \cdot (\frac{3}{4})^2 - 3 \cdot \frac{3}{4} + 1 = -\frac{1}{8}$$

Top($\frac{3}{4}$, $-\frac{1}{8}$).

x	-2	-1	0	$\frac{3}{4}$	1	2	3
y	15	6	1	$-\frac{1}{8}$	0	3	10

$$y = -3x^2 - 9x + 6$$

Snijpunt y - as:

$$y = -3 \cdot 0^2 - 9 \cdot 0 + 6 = 6$$

(0, 6)

$$-3x^2 - 9x + 6 = 6$$

$$-3x^2 - 9x = 0$$

$$x^2 + 3x = 0$$

$$x(x + 3) = 0$$

$$x = 0 \text{ of } x = -3$$

Symmetrieas:

$$x = \frac{0 + (-3)}{2} = -1\frac{1}{2}$$

$$y = -3 \cdot (-1\frac{1}{2})^2 - 9 \cdot (-1\frac{1}{2}) + 6 = 12\frac{3}{4}$$

Top($-1\frac{1}{2}$, $12\frac{3}{4}$).

x	-4	-3	-2	$-1\frac{1}{2}$	-1	0	1
y	-6	6	12	$12\frac{3}{4}$	12	6	-6

$$y = x(x - 8) + 2$$

Snijpunt y - as:

$$y = 0 \cdot (0 - 8) + 2 = 2$$

(0, 2)

$$x(x - 8) + 2 = 2$$

$$x(x - 8) = 0$$

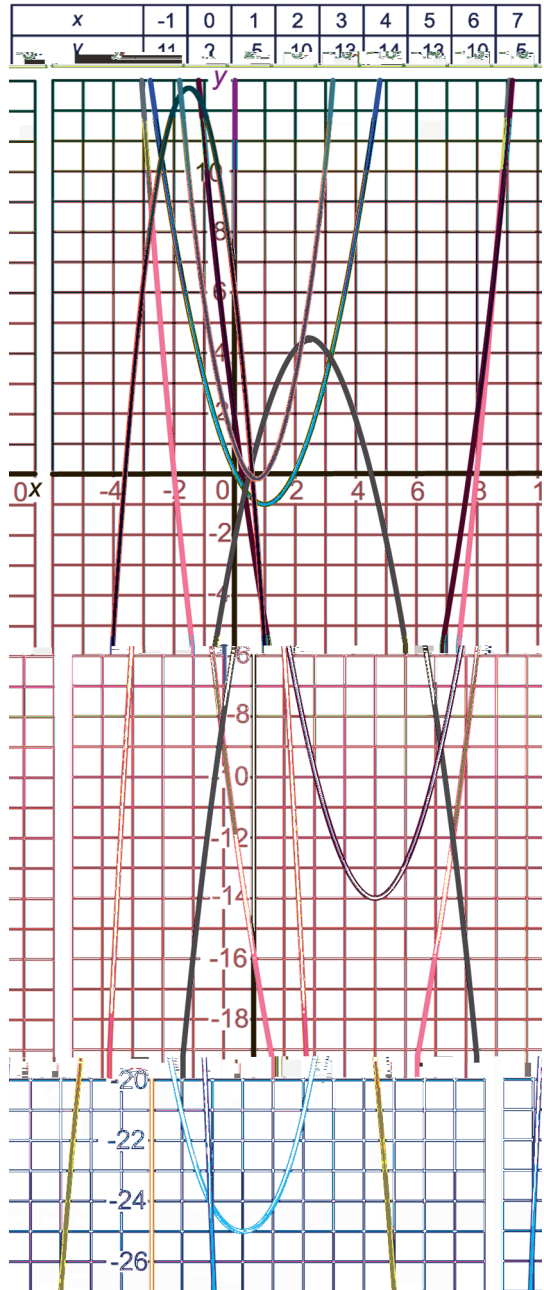
$$x = 0 \text{ of } x = 8$$

Symmetrieas:

$$x = \frac{0 + 8}{2} = 4$$

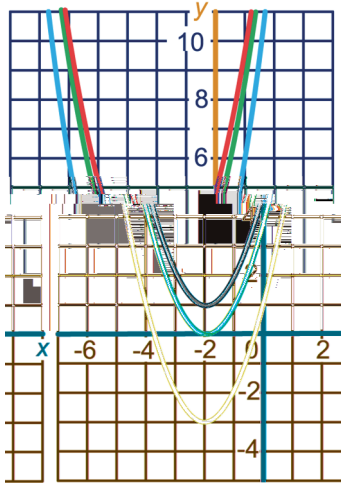
$$y = 4 \cdot (4 - 8) + 2 = -14$$

Top(4, -14).

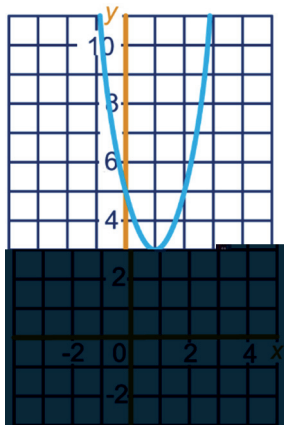


- 19 a (0, 0)
b (0, 1); (0, -3)
c $y \geq 0$
d (-2, 0)
e (-2, 1); (-2, -3)
f

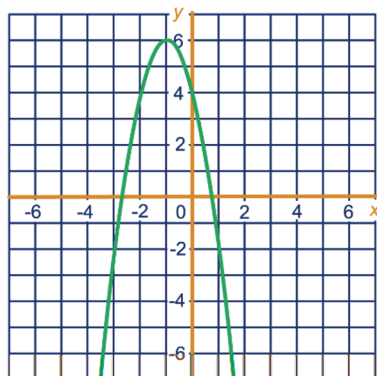
x	-5	-4	-3	-2	-1	0	1
$y = (x + 2)^2$	9	4	1	0	1	4	9
$y = (x + 2)^2 + 1$	10	5	2	1	2	5	10
$y = (x + 2)^2 - 3$	6	1	-2	-3	-2	1	6



- 20 a** Omdat $2(x-1)^2$ voor elke waarde van x , 0 of meer is. Dus $2(x-1)^2 + 3$ is minimaal 3.
b $y \geq 3$
c Een dalparabool, omdat $2(x-1)^2$ minimaal 0 is.
d bijv. $y = 2(x-1)^2$
e Door bijv. van $2(x-1)^2$ een getal af te trekken.
f vergelijking symmetrieas: $x = 1$
g Top(1, 3).
h



- 21 a** Een bergparabool, omdat $-2(x+1)^2$ maximaal 0 is,
b Twee nulpunten, omdat de top op hoogte 6 ligt en het een bergparabool is.
c vergelijking symmetrieas: $x = -1$
d Top(-1, 6).
e



- 22 a** Dalparabool als $c > 0$,
 een bergparabool als $c < 0$.
b (a, b)
23 (-2, -3) en (9, 19)

29.4 TOEPASSINGEN

- 24 a** $x - \frac{1}{100}x^2 = 0$
 $100x - x^2 = 0$
 $x(100 - x) = 0$
 $x = 0$ of $x = 100$
 Dus de kogel legt 100 meter af.
b Vanwege symmetrie wordt de grootste hoogte bereikt als $x = 50$.
 Dan $y = 50 - \frac{1}{100} \cdot 50^2 = 25$, dus 25 meter.
- 25 a** $C = 3 \cdot 1\frac{1}{2} \cdot 10 = 45$
b $C = x(6 - 2x) \cdot 10 = 60x - 20x^2$
c een bergparabool
d Nulpunten:
 $60x - 20x^2 = 0$
 $20x(3 - x) = 0$
 $x = 0$ of $x = 3$
 Top ligt bij $x = 1\frac{1}{2} \Rightarrow y = 60 \cdot 1\frac{1}{2} - 2 \cdot (1\frac{1}{2})^2 = 45$
 Top($1\frac{1}{2}$, 45).
e Bij hoogte $1\frac{1}{2}$ dm, de capaciteit is dan 45 liter.
- 26** $y = cx^2$
 $62,5 = c \cdot 250^2$ (invullen het punt (250; 62,5))
 $62,5 = 62.500c$
 $\frac{1}{1000} = c$
 Vergelijking parabool: $y = \frac{1}{1000}x^2$
- 27 a** $(10 + 2)^2 - 10 - 10 = 124$ stippen
b $(n + 2)^2 - 2n = n^2 + 2n + 4$
c $n^2 + 2n + 4 = 10204$
 $n^2 + 2n - 10200 = 0$
 $(n - 100)(n + 102) = 0$
 $n = 100$ of $n = -102$
 Alleen $n = 100$ voldoet, omdat $n > 0$ moet zijn.
d $n^2 + 2n + 4 + 43 = (n + 1)^2 + 2(n + 1) + 4$
 $n^2 + 2n + 47 = n^2 + 4n + 7$
 $40 = 2n$
 $20 = n$
 Rangnummer is 20. Dus rangnummer 21 heeft 43 stippen meer dan rangnummer 20.

- 28 a** $(60 - 2x)^2 = 1000$
 $60 - 2x = \sqrt{1000}$ of $60 - 2x = -\sqrt{1000}$
 $2x = 60 - \sqrt{1000}$ of $2x = 60 + \sqrt{1000}$
 $2x = 60 - 10\sqrt{10}$ of $2x = 60 + 10\sqrt{10}$
 $x = 30 - 5\sqrt{10}$ of $x = 30 + 5\sqrt{10}$
 $x \approx 14,19$ cm of $x \approx 45,81$ cm
Alleen $x = 30 - 5\sqrt{10} \approx 14,19$ cm voldoet,
omdat $2x < 60$, dus $x < 30$ moet zijn.
- b** $(60 - 2x)^2 = 4 \cdot x \cdot (60 - 2x)$
 $3600 - 240x + 4x^2 = 240x - 8x^2$
 $12x^2 - 480x + 3600 = 0$
 $x^2 - 40x + 300 = 0$
 $(x - 30)(x - 10) = 0$
 $x = 30$ of $x = 10$
Alleen $x = 10$ cm voldoet, omdat $x < 30$ moet zijn.

- 29 a** $-t^2 + 4t + 21 = 21$
 $-t^2 + 4t = 0$
 $t^2 - 4t = 0$
 $t(t - 4) = 0$
 $t = 0$ of $t = 4$
Dus na 4 sec. is het steentje weer op
dezelfde hoogte.
- b** $-t^2 + 4t + 21 = 0$
 $t^2 - 4t - 21 = 0$
 $(t - 7)(t + 3) = 0$
 $t = 7$ of $t = -3$
Dus na 7 sec. bereikt het steentje het water.
- c** vergelijking symmetrieas: $t = \frac{7-3}{2} = 2$
 $h = -2^2 + 4 \cdot 2 + 21 = 25$
Op 2 sec. bereikt het steentje de maximale
hoogte van 25 m.

- 30 a** $2l + 2b = 44 \Rightarrow l + b = 22$
- b** $l + b = 22 \Rightarrow l = 22 - b$
 $(22 - b) \cdot b = 120$
 $22b - b^2 = 120$
 $b^2 - 22b + 120 = 0$
 $(b - 12)(b - 10) = 0$
 $b = 12$ of $b = 10$
Als $b = 12$, dan $l = 22 - 12 = 10$.
Als $b = 10$, dan $l = 22 - 10 = 12$.
De rechthoek is 12 bij 10 of de rechthoek is
10 bij 12.

- c** Stelsel:
 $\begin{cases} l \cdot b = 720 \\ l = 5b \end{cases}$
Vergelijking:
 $5b \cdot b = 720$
 $5b^2 = 720$
 $b^2 = 144$
 $b = 12$ of $b = -12$
Alleen $b = 12$, voldoet omdat $b > 0$ moet zijn.
Dan $l = 5 \cdot 12 = 60$.
Dus de breedte is 12 en de lengte 60.

29.5 VERGELIJKINGEN OPLOSSEN

- 31** $(x - 3)^2 = 100$
 $x - 3 = 10$ of $x - 3 = -10$
 $x = 13$ of $x = -7$
- $(x + \frac{1}{2})^2 = 25$
 $x + \frac{1}{2} = 5$ of $x + \frac{1}{2} = -5$
 $x = 4\frac{1}{2}$ of $x = -5\frac{1}{2}$
- $(2x + 1)^2 = 36$
 $2x + 1 = 6$ of $2x + 1 = -6$
 $2x = 5$ of $2x = -7$
 $x = 2\frac{1}{2}$ of $x = -3\frac{1}{2}$
- $(x - 3)^2 = 13$
 $x - 3 = \sqrt{13}$ of $x - 3 = -\sqrt{13}$
 $x = 3 + \sqrt{13}$ of $x = 3 - \sqrt{13}$
- $(x + \frac{1}{2})^2 = 48$
 $x + \frac{1}{2} = \sqrt{48} = 4\sqrt{3}$ of $x + \frac{1}{2} = -\sqrt{48} = -4\sqrt{3}$
 $x = -\frac{1}{2} + 4\sqrt{3}$ of $x = -\frac{1}{2} - 4\sqrt{3}$
- $(2x + 1)^2 = 68$
 $2x + 1 = \sqrt{68} = 2\sqrt{17}$ of $2x + 1 = -\sqrt{68} = -2\sqrt{17}$
 $2x = -1 + 2\sqrt{17}$ of $2x = -1 - 2\sqrt{17}$
 $x = -\frac{1}{2} + \sqrt{17}$ of $x = -\frac{1}{2} - \sqrt{17}$
- 32** 25 ; 5 7 ; x ; $3\frac{1}{2}$
12 ; 36 ; x 4 ; 16
16 ; x ; 8 $5\frac{1}{2}$; $30\frac{1}{4}$
 $20\frac{1}{4}$; $4\frac{1}{2}$ 3 ; 9
11 ; $30\frac{1}{4}$; x

33 $x^2 + 10x = 90$
 $x^2 + 10x + 25 = 90 + 25$
 $(x+5)^2 = 115$
 $x+5 = \sqrt{115}$ of $x+5 = -\sqrt{115}$
 $x = -5 + \sqrt{115}$ of $x = -5 - \sqrt{115}$

$x^2 + 7x + 1 = 0$
 $x^2 + 7x + 1 + 11\frac{1}{4} = 11\frac{1}{4}$
 $x^2 + 7x + 12\frac{1}{4} = 11\frac{1}{4}$
 $(x + 3\frac{1}{2})^2 = 11\frac{1}{4} = \frac{45}{4}$
 $x + 3\frac{1}{2} = \sqrt{\frac{45}{4}} = \frac{1}{2}\sqrt{45} = 1\frac{1}{2}\sqrt{5}$ of
 $x + 3\frac{1}{2} = -1\frac{1}{2}\sqrt{5}$
 $x = -3\frac{1}{2} + 1\frac{1}{2}\sqrt{5}$ of $x = -3\frac{1}{2} - 1\frac{1}{2}\sqrt{5}$

$x^2 + 10x + 22 = 0$
 $x^2 + 10x + 22 + 3 = 3$
 $x^2 + 10x + 25 = 3$
 $(x+5)^2 = 3$
 $x+5 = \sqrt{3}$ of $x+5 = -\sqrt{3}$
 $x = -5 + \sqrt{3}$ of $x = -5 - \sqrt{3}$

$x^2 - 11x = -7$
 $x^2 - 11x + 30\frac{1}{4} = -7 + 30\frac{1}{4}$
 $(x - 5\frac{1}{2})^2 = 23\frac{1}{4} = \frac{93}{4}$
 $x - 5\frac{1}{2} = \sqrt{\frac{93}{4}} = \frac{1}{2}\sqrt{93}$ of $x - 5\frac{1}{2} = -\frac{1}{2}\sqrt{93}$
 $x = 5\frac{1}{2} + \frac{1}{2}\sqrt{93}$ of $x = 5\frac{1}{2} - \frac{1}{2}\sqrt{93}$

$x^2 - 12x = -23$
 $x^2 - 12x + 36 = -23 + 36$
 $(x-6)^2 = 13$
 $x-6 = \sqrt{13}$ of $x-6 = -\sqrt{13}$
 $x = 6 + \sqrt{13}$ of $x = 6 - \sqrt{13}$

$x^2 - 5x - 1 = 0$
 $x^2 - 5x - 1 + 7\frac{1}{4} = 7\frac{1}{4}$
 $x^2 - 5x + 6\frac{1}{4} = 7\frac{1}{4}$
 $(x - 2\frac{1}{2})^2 = 7\frac{1}{4} = \frac{29}{4}$
 $x - 2\frac{1}{2} = \sqrt{\frac{29}{4}} = \frac{1}{2}\sqrt{29}$ of $x - 2\frac{1}{2} = -\frac{1}{2}\sqrt{29}$
 $x = 2\frac{1}{2} + \frac{1}{2}\sqrt{29}$ of $x = 2\frac{1}{2} - \frac{1}{2}\sqrt{29}$

$x^2 = x + 3$
 $x^2 - x = 3$
 $x^2 - x + \frac{1}{4} = 3 + \frac{1}{4}$
 $(x - \frac{1}{2})^2 = 3\frac{1}{4} = \frac{13}{4}$
 $x - \frac{1}{2} = \sqrt{\frac{13}{4}} = \frac{1}{2}\sqrt{13}$ of $x - \frac{1}{2} = -\frac{1}{2}\sqrt{13}$
 $x = \frac{1}{2} + \frac{1}{2}\sqrt{13}$ of $x = \frac{1}{2} - \frac{1}{2}\sqrt{13}$

$x^2 - 9x - 3 = 0$
 $x^2 - 9x - 3 + 23\frac{1}{4} = 23\frac{1}{4}$
 $x^2 - 9x + 20\frac{1}{4} = 23\frac{1}{4}$
 $(x - 4\frac{1}{2})^2 = 23\frac{1}{4} = \frac{93}{4}$
 $x - 4\frac{1}{2} = \sqrt{\frac{93}{4}} = \frac{1}{2}\sqrt{93}$ of $x - 4\frac{1}{2} = -\frac{1}{2}\sqrt{93}$
 $x = 4\frac{1}{2} + \frac{1}{2}\sqrt{93}$ of $x = 4\frac{1}{2} - \frac{1}{2}\sqrt{93}$

34 a $a = 2$, $b = 12$ en $c = 6$.

b $x = \frac{-12 + \sqrt{12^2 - 4 \cdot 2 \cdot 6}}{2 \cdot 2} = \frac{-12 + \sqrt{96}}{4} = \frac{-12 + 4\sqrt{6}}{4} = -3 + \sqrt{6}$

$x = \frac{-12 - \sqrt{12^2 - 4 \cdot 2 \cdot 6}}{2 \cdot 2} = \frac{-12 - \sqrt{96}}{4} = \frac{-12 - 4\sqrt{6}}{4} = -3 - \sqrt{6}$

35 Dan staat er een lineaire vergelijking.

36 $2x^2 - 3x - 35 = 0$

$$\left. \begin{array}{l} a = 2 \\ b = -3 \\ c = -35 \end{array} \right\} \begin{array}{l} D = 9 - 4 \cdot 2 \cdot -35 = 289, \\ \sqrt{D} = 17 \end{array}$$

$x = \frac{3+17}{4} = 5$ of $x = \frac{3-17}{4} = -3\frac{1}{2}$

$2x^2 + 4x - 1 = 0$

$$\left. \begin{array}{l} a = 2 \\ b = 4 \\ c = -1 \end{array} \right\} \begin{array}{l} D = 16 - 4 \cdot 2 \cdot -1 = 24, \\ \sqrt{D} = \sqrt{24} = 2\sqrt{6} \end{array}$$

$x = \frac{-4+2\sqrt{6}}{4} = -1 + \frac{1}{2}\sqrt{6}$ of $x = \frac{-4-2\sqrt{6}}{4} = -1 - \frac{1}{2}\sqrt{6}$

$7x^2 - 6x + 2 = 0$

$$\left. \begin{array}{l} a = 7 \\ b = -6 \\ c = 2 \end{array} \right\} D = 36 - 4 \cdot 7 \cdot 2 = -20$$

$D < 0$, dus géén oplossingen

$$\frac{1}{2}x^2 - 3x - 4\frac{1}{2} = 0$$

$$\left. \begin{array}{l} a = \frac{1}{2} \\ b = -3 \\ c = -4\frac{1}{2} \end{array} \right\} D = 9 - 4 \cdot \frac{1}{2} \cdot -4\frac{1}{2} = 18, \\ \sqrt{D} = \sqrt{18} = 3\sqrt{2}$$

$$x = \frac{3+3\sqrt{2}}{1} = 3+3\sqrt{2} \text{ of } x = \frac{3-3\sqrt{2}}{1} = 3-3\sqrt{2}$$

$$4x = 1 + 4x^2$$

$$4x^2 - 4x + 1 = 0$$

$$\left. \begin{array}{l} a = 4 \\ b = -4 \\ c = 1 \end{array} \right\} D = 16 - 4 \cdot 4 \cdot 1 = 0$$

$$x = -\frac{-4}{2} = \frac{1}{2}$$

$$(x-3)^2 = 5 - 3x$$

$$x^2 - 6x + 9 = 5 - 3x$$

$$x^2 - 3x + 4 = 0$$

$$\left. \begin{array}{l} a = 1 \\ b = -3 \\ c = 4 \end{array} \right\} D = 9 - 4 \cdot 1 \cdot 4 = -7$$

$D < 0$, dus geen oplossingen

$$5x - 3x^2 = 0$$

$$\left. \begin{array}{l} a = -3 \\ b = 5 \\ c = 0 \end{array} \right\} D = 25 - 4 \cdot -3 \cdot 0 = 25, \\ \sqrt{D} = 5$$

$$x = \frac{-5+5}{-6} = 0 \text{ of } x = \frac{-5-5}{-6} = \frac{-10}{-6} = 1\frac{2}{3}$$

SUPER OPGAVEN

15 a $\frac{18}{45} = \frac{12}{y}$

$$18y = 540$$

$$y = 30$$

$$O = 12 \cdot (45 - 30) = 180$$

b $\frac{18}{45} = \frac{x}{y}$

$$18y = 45x$$

$$y = 2\frac{1}{2}x$$

c Breedte van de rechthoek is

$$45 - y = 45 - 2\frac{1}{2}x$$

$$O = x \cdot (45 - 2\frac{1}{2}x) = 45x - 2\frac{1}{2}x^2$$

d $45x - 2\frac{1}{2}x^2 = 0$

$$2\frac{1}{2}x(18 - x) = 0$$

$$x = 0 \text{ of } x = 18$$

De oppervlakte is maximaal als $x = 9$.

e De oppervlakte is dan $45 \cdot 9 - 2\frac{1}{2} \cdot 9^2 = 202\frac{1}{2}$.

23 Als de top op de y-as ligt, dan zijn (-2,4) en (3,6) ook punten van de parabool. Dus dan moet het een dalparabool zijn.

29 a Oppervlakte vierkant is $6 \cdot 6 = 36$, totale oppervlakte wit is $x \cdot x + (6-x)(6-x) = 2x^2 - 12x + 36$, oppervlakte oker is $36 - (2x^2 - 12x + 36) = -2x^2 + 12x$.

b $-2x^2 + 12x = 0$

$$-2x(x-6) = 0$$

$$x = 0 \text{ of } x = 6$$

De nulpunten zijn 0 en 6.

c Als $x = 3$, dan oppervlakte is $-2 \cdot 3^2 + 12 \cdot 3 = 18$.

30 a Stelsel:

$$\begin{cases} l \cdot b = 400 \\ 2l + 2b - 4 = 96 \end{cases}$$

b $2l + 2b - 4 = 96$

$$2l + 2b = 100$$

$$l + b = 50$$

$$l = 50 - b$$

Vergelijking:

$$(50 - b) \cdot b = 400$$

$$50b - b^2 = 400$$

$$b^2 - 50b + 400 = 0$$

$$(b-40)(b-10) = 0$$

$$b = 40 \text{ of } b = 10$$

Als $b = 40$, dan $l = 400 : 40 = 10$.

Als $b = 10$, dan $l = 400 : 10 = 40$.

De afmetingen zijn 40 bij 10 of 10 bij 40.

29.8 EXTRA OPGAVEN

1 a $y = x^2 + x$

Nulpunten:

$$x^2 + x = 0$$

$$x(x+1) = 0$$

$$x = 0 \text{ of } x = -1$$

Snijpunt y - as:

$$y = 0^2 + 0 = 0$$

(0, 0)

Symmetrieas:

$$x = \frac{0-1}{2} = -\frac{1}{2}$$

$$y = \left(-\frac{1}{2}\right)^2 - \frac{1}{2} = -\frac{1}{4}$$

Top $\left(-\frac{1}{2}, -\frac{1}{4}\right)$.

x	-3	-2	-1	$-\frac{1}{2}$	0	1	2
y	7	2	0	$-\frac{1}{4}$	0	2	7

$$y = x^2 - 7x$$

Nulpunten:

$$x^2 - 7x = 0$$

$$x(x-7) = 0$$

$$x = 0 \text{ of } x = 7$$

Snijpunt y – as:

$$y = 0^2 - 7 \cdot 0 = 0$$

(0, 0)

Symmetrieas:

$$x = \frac{0+7}{2} = 3\frac{1}{2}$$

$$y = \left(3\frac{1}{2}\right)^2 - 7 \cdot 3\frac{1}{2} = -12\frac{1}{4}$$

Top($3\frac{1}{2}$, $-12\frac{1}{4}$).

x	-1	0	1	2	3	$3\frac{1}{2}$	4	5	6	7
y	8	0	-6	-10	-12	$-12\frac{1}{4}$	-12	-10	-6	0

$$y = -3x^2$$

Nulpunten:

$$-3x^2 = 0$$

$$x = 0$$

Snijpunt y – as:

$$y = -3 \cdot 0^2$$

(0, 0)

Symmetrieas:

$$x = \frac{0+0}{2} = 0$$

$$y = -3 \cdot 0^2$$

Top(0, 0).

x	-3	-2	-1	0	1	2
y	-27	-12	-3	0	-3	-12

$$y = (x+2)^2 - 3$$

Nulpunten:

$$(x+2)^2 - 3 = 0$$

$$(x+2)^2 = 3$$

$$x+2 = \sqrt{3} \text{ of } x+2 = -\sqrt{3}$$

$$x = -2 + \sqrt{3} \text{ of } x = -2 - \sqrt{3}$$

Snijpunt y – as:

$$y = (0+2)^2 - 3 = 1$$

(0, 1)

Top(-2, -3).

Symmetrieas:

$$x = -2$$

x	-5	-4	-3	-2	-1	0	1
y	6	1	-2	-3	-2	1	6

$$y = -\frac{1}{2}(x-1)^2 + 8$$

Nulpunten:

$$-\frac{1}{2}(x-1)^2 + 8 = 0$$

$$-\frac{1}{2}(x-1)^2 = -8$$

$$(x-1)^2 = 16$$

$$x-1 = 4 \text{ of } x-1 = -4$$

$$x = 5 \text{ of } x = -3$$

Snijpunt y – as:

$$y = -\frac{1}{2}(0-1)^2 + 8 = 7\frac{1}{2}$$

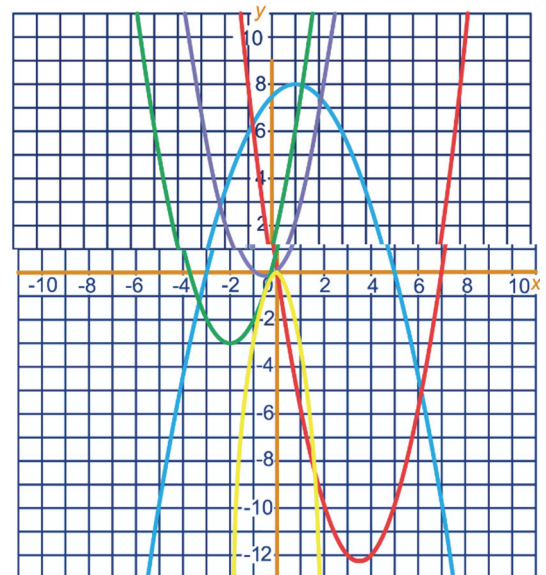
(0, $7\frac{1}{2}$)

Top(1, 8).

Symmetrieas:

$$x = \frac{-3+5}{2} = 1$$

x	-3	-2	-1	0	1	2	3	4	5	6
y	0	$3\frac{1}{2}$	6	$7\frac{1}{2}$	8	$7\frac{1}{2}$	6	$3\frac{1}{2}$	0	$-4\frac{1}{2}$



2

$$y = x^2 + 12x$$

Nulpunten:

$$x^2 + 12x = 0$$

$$x(x+12) = 0$$

$$x = 0 \text{ of } x = -12$$

Symmetrieas:

$$x = \frac{0-12}{2} = -6$$

$$y = (-6)^2 + 12 \cdot -6 = -36$$

Top(-6, -36).

$$y = 2x^2 - 5x$$

Nulpunten:

$$2x^2 - 5x = 0$$

$$2x(x - 2\frac{1}{2}) = 0$$

$$x = 0 \text{ of } x = 2\frac{1}{2}$$

Symmetrieas:

$$x = \frac{0+2\frac{1}{2}}{2} = 1\frac{1}{4}$$

$$y = 2 \cdot (1\frac{1}{4})^2 - 5 \cdot 1\frac{1}{4} = -3\frac{1}{8}$$

Top($1\frac{1}{4}$, $-3\frac{1}{8}$).

$$y = x^2 + 3x + 2$$

Nulpunten:

$$x^2 + 3x + 2 = 0$$

$$(x+1)(x+2) = 0$$

$$x = -1 \text{ of } x = -2$$

Symmetrieas:

$$x = \frac{-1-2}{2} = -1\frac{1}{2}$$

$$y = (-1\frac{1}{2})^2 + 3 \cdot -1\frac{1}{2} + 2 = -\frac{1}{4}$$

Top($-1\frac{1}{2}$, $-\frac{1}{4}$).

$$y = -x^2 + 4x + 6$$

$$-x^2 + 4x + 6 = 6$$

$$-x^2 + 4x = 0$$

$$-x(x-4) = 0$$

$$x = 0 \text{ of } x = 4$$

Symmetrieas:

$$x = \frac{0+4}{2} = 2$$

$$y = -2^2 + 4 \cdot 2 + 6 = 10$$

Top(2, 10).

3 a $y = cx^2$
 $3 = c \cdot 4^2$ (invullen het punt (4,3))

$$3 = 16c$$

$$\frac{3}{16} = c$$

Vergelijking parabool: $y = \frac{3}{16}x^2$

b $x = 3$ of $x = -3 \Rightarrow y = \frac{3}{16} \cdot 3^2 = 1\frac{11}{16}$

Dus $(3, 1\frac{11}{16})$ en $(-3, 1\frac{11}{16})$.

- 4** 100 ; 10
18 ; 81 ; x
 $42\frac{1}{4}$; $6\frac{1}{2}$
5 ; $6\frac{1}{4}$; x
6 ; 36
1 ; 1

5 $14 = x(x-5)$

$$x^2 - 5x - 14 = 0$$

$$(x-7)(x+2) = 0$$

$$x = 7 \text{ of } x = -2$$

$$2x^2 + x = 5x + 8$$

$$2x^2 - 4x - 8 = 0$$

$$x^2 - 2x - 4 = 0$$

$$x^2 - 2x - 4 + 5 = 5$$

$$x^2 - 2x + 1 = 5$$

$$(x-1)^2 = 5$$

$$x-1 = \sqrt{5} \text{ of } x-1 = -\sqrt{5}$$

$$x = 1 + \sqrt{5} \text{ of } x = 1 - \sqrt{5}$$

$$25 = 4(x+1)^2$$

$$(x+1)^2 = 6\frac{1}{4}$$

$$x+1 = 2\frac{1}{2} \text{ of } x+1 = -2\frac{1}{2}$$

$$x = 1\frac{1}{2} \text{ of } x = -3\frac{1}{2}$$

$$(x+1)^2 + (x+3)^2 = 4x^2$$

$$2x^2 + 8x + 10 = 4x^2$$

$$2x^2 - 8x - 10 = 0$$

$$x^2 - 4x - 5 = 0$$

$$(x-5)(x+1) = 0$$

$$x = 5 \text{ of } x = -1$$

$$x^2 - 3x = 2x^2 + x + 1$$

$$x^2 + 4x + 1 = 0$$

$$x^2 + 4x + 1 + 3 = 3$$

$$x^2 + 4x + 4 = 3$$

$$(x+2)^2 = 3$$

$$x+2 = \sqrt{3} \text{ of } x+2 = -\sqrt{3}$$

$$x = -2 + \sqrt{3} \text{ of } x = -2 - \sqrt{3}$$

$$x^2 + 5x + 3 = 0$$

$$x^2 + 5x + 3 + 3\frac{1}{4} = 3\frac{1}{4}$$

$$x^2 + 5x + 6\frac{1}{4} = 3\frac{1}{4}$$

$$(x+2\frac{1}{2})^2 = 3\frac{1}{4} = \frac{13}{4}$$

$$x+2\frac{1}{2} = \sqrt{\frac{13}{4}} = \frac{1}{2}\sqrt{13} \text{ of } x+2\frac{1}{2} = -\frac{1}{2}\sqrt{13}$$

$$x = -2\frac{1}{2} + \frac{1}{2}\sqrt{13} \text{ of } x = -2\frac{1}{2} - \frac{1}{2}\sqrt{13}$$

- 6 a Oppervlakte grasveld is $4 \cdot 4 = 16$,
oppervlakte border is $3 \cdot 4x + 2 \cdot x^2 = 12x + 2x^2$.

$$12x + 2x^2 = 16$$

$$x^2 + 6x - 8 = 0$$

$$x^2 + 6x - 8 + 17 = 17$$

$$x^2 + 6x + 9 = 17$$

$$(x+3)^2 = 17$$

$$x+3 = \sqrt{17} \quad \text{of} \quad x+3 = -\sqrt{17}$$

$$x = -3 + \sqrt{17} \quad \text{of} \quad x = -3 - \sqrt{17}$$

Dus $x = -3 + \sqrt{17}$ m, omdat $x > 0$ moet zijn.

- b $12x + 2x^2 = 2 \cdot 16$

$$x^2 + 6x - 16 = 0$$

$$(x+8)(x-2) = 0$$

$$x = -8 \quad \text{of} \quad x = 2$$

Dus $x = 2$ m, omdat $x > 0$ moet zijn.

- 7 $3x^2 + 10x + 3 = 0$

$$\left. \begin{array}{l} a = 3 \\ b = 10 \\ c = 3 \end{array} \right\} \begin{array}{l} D = 100 - 4 \cdot 3 \cdot 3 = 64, \\ \sqrt{D} = 8 \end{array}$$

$$x = \frac{-10+8}{6} = -\frac{1}{3} \quad \text{of} \quad x = \frac{-10-8}{6} = -3$$

$$2x^2 = 5x - 3$$

$$2x^2 - 5x + 3 = 0$$

$$\left. \begin{array}{l} a = 2 \\ b = -5 \\ c = 3 \end{array} \right\} \begin{array}{l} D = 25 - 4 \cdot 2 \cdot 3 = 1, \\ \sqrt{D} = 1 \end{array}$$

$$x = \frac{5+1}{4} = 1\frac{1}{2} \quad \text{of} \quad x = \frac{5-1}{2} = 2$$

$$x^2 - 8x = -22$$

$$x^2 - 8x + 22 = 0$$

$$\left. \begin{array}{l} a = 1 \\ b = -8 \\ c = 22 \end{array} \right\} D = 64 - 4 \cdot 1 \cdot 22 = -24$$

$D < 0$, dus geen oplossingen

$$-5x^2 + 4x - \frac{4}{5} = 0$$

$$\left. \begin{array}{l} a = -5 \\ b = 4 \\ c = -\frac{4}{5} \end{array} \right\} D = 16 - 4 \cdot (-5) \cdot (-\frac{4}{5}) = 0$$

$$x = -\frac{4}{-10} = \frac{2}{5}$$

- 8 a Oppervlakte driehoek is $\frac{1}{2} \cdot x(8-x) = 4x - \frac{1}{2}x^2$.

b $\frac{1}{4}$ deel ; $\frac{1}{4} \cdot 8 \cdot 8 = 16$

c $2 \cdot x(8-x) = 16$

$$2x^2 - 16x + 16 = 0$$

$$x^2 - 8x + 8 = 0$$

$$\left. \begin{array}{l} a = 1 \\ b = -8 \\ c = 8 \end{array} \right\} \begin{array}{l} D = 64 - 4 \cdot 1 \cdot 8 = 32, \\ \sqrt{D} = \sqrt{32} = 4\sqrt{2} \end{array}$$

$$x = \frac{8+4\sqrt{2}}{2} = 4 + 2\sqrt{2} \text{ cm} \quad \text{of} \quad x = 4 - 2\sqrt{2} \text{ cm}$$

- 9 a hoogte = x , breedte = $x + 3$, lengte = $x + 4$

b oppervlakte =

$$2(x(x+4) + x(x+3) + (x+4)(x+3)) =$$

$$2(3x^2 + 14x + 12) = 6x^2 + 28x + 24$$

$$6x^2 + 28x + 24 = 162$$

$$6x^2 + 28x - 138 = 0$$

$$\left. \begin{array}{l} a = 6 \\ b = 28 \\ c = -138 \end{array} \right\} \begin{array}{l} D = 784 - 4 \cdot 6 \cdot (-138) = 4096, \\ \sqrt{D} = 64 \end{array}$$

$$x = \frac{-28+64}{12} = 3 \quad \text{of} \quad x = \frac{-28-64}{12} = -7\frac{2}{3}$$

Alleen $x = 3$ voldoet, omdat $x > 0$ moet zijn.

- 10 a $50t - 5t^2 = 0$

$$5t(10-t) = 0$$

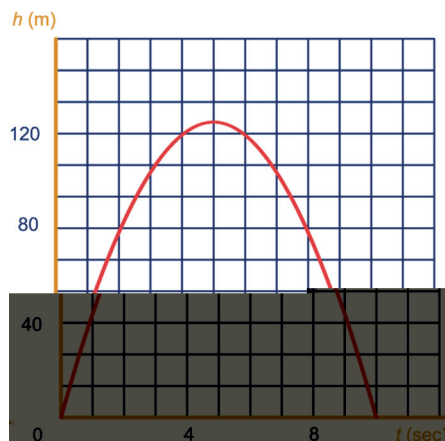
$$t = 0 \quad \text{of} \quad t = 10$$

Dus de vlucht duurt $10 - 0 = 10$ sec.

- b Maximale hoogte wordt bereikt na 5 sec.,

$$h = 50 \cdot 5 - 5 \cdot 5^2 = 250 - 125 = 125 \text{ m.}$$

c



- d $50t - 5t^2 > 113,75$

$$0 > 5t^2 - 50t + 113,75$$

$$t^2 - 10t + 22,75 < 0$$

$$(t-3,5)(t-6,5) < 0$$

$$3,5 < t < 6,5$$

Dus tussen de 3,5 en 6,5 sec. is de hoogte van de vuurpijl meer dan 113,75 m.