

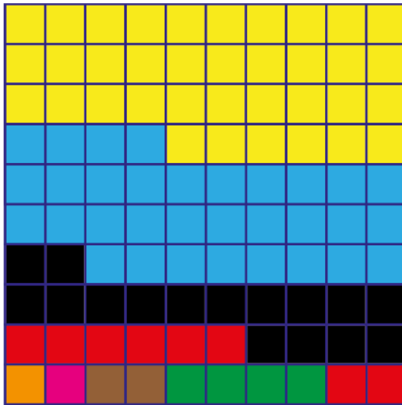
Hoofdstuk 11 MACHTEN

11.0 INTRO

...
 $2 \cdot 2 \cdot 2 \cdot 2 = 16$ stukjes
 $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 64$ stukjes

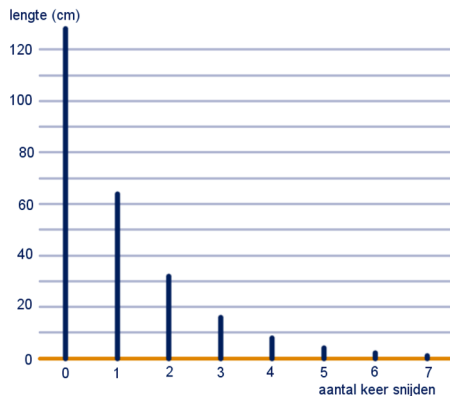
11.1 OVERAL MACHTEN

?



1	2	3	4	5	6	7	8
1	2	4	8	16	32	64	(128, maar past niet)

Na 7 dagen.



0	1	2	3	4	5	6	7
128	64	32	16	8	4	2	1
1	2	4	8	16	32	64	128

0	1	2	3	4	5
1	2	4	8	16	32

In minder dan 7 uur.

1	2	4	8	16	32	64
53	106	212	424	848	1696	3392

$$13 \cdot 53 = (1 + 4 + 8) \cdot 53 = 1 \cdot 53 + 4 \cdot 53 + 8 \cdot 53$$

$$17 \cdot 53 = 1 \cdot 53 + 16 \cdot 53 = 53 + 848 = 901$$

$$35 \cdot 53 = 1 \cdot 53 + 2 \cdot 53 + 32 \cdot 53 = 53 + 106 + 1696 = 1855$$

Verdubbelingstabel van 321:

1	2	4	8	16	32
321	642	1284	2568	5136	10.272

$$9 \cdot 321 = 1 \cdot 321 + 8 \cdot 321 = 321 + 2568 = 2889$$

$$28 \cdot 321 = 4 \cdot 321 + 8 \cdot 321 + 16 \cdot 321 = 1284 + 2568 + 5136 = 8988$$

$$51 \cdot 321 = 1 \cdot 321 + 2 \cdot 321 + 16 \cdot 321 + 32 \cdot 321 = 321 + 642 + 5136 + 10.272 = 16.371$$

$2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$; twee tot de macht vijf; 32
 $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$; twee tot de macht tien; 1024

2	3	4	5	6	7	8	9	10
4	8	16	32	64	128	256	512	1024

1%
1

$10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 = 10.000.000$
 12 cijfers (namelijk elf 0'en en één 1)
 $n + 1$ cijfers

$$5000 + 200 + 70 + 1 = 5271$$

$$70.000.000 + 500.000 + 3000 + 10 = 70.503.010$$

$$3000 + 4600 + 110 = 7710$$

$$1000 \cdot (2000 + 200 + 20 + 2) = 2.222.000$$

$$100 \cdot (100 \cdot 101 + 1) + 1 = 100 \cdot 10.101 + 1 = 1.010.101$$

10^5 ; 10^7 ; 10^6

duizend; tienduizend; honderdduizend;
 honderd miljoen; tien miljard; tien biljoen

$$2^2 \cdot 2^6 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 2^8$$

$$2^5 \cdot 2^7 = 2^{5+7} = 2^{12}$$

$$2^4 \cdot 2^8 = 2^{4+8} = 2^{12}$$

$$2^5 \cdot 2^6 = 2^{5+6} = 2^{11}$$

$$2^5 \cdot 2^0 = 2^{5+0} = 2^5$$

$$2^5 \cdot 2^{\dots} = 2^{20} \rightarrow 2^{20-5} = 2^{15}$$

$$2 \cdot 2^5 \cdot 2^7 = 2^{1+5+7} = 2^{13}$$

$$2^3 \cdot 2^4 \cdot 2^8 = 2^{3+4+8} = 2^{15}$$

$$2^4 \cdot 2^5 \cdot 2^6 = 2^{4+5+6} = 2^{15}$$

$$2^5 \cdot 2^0 \cdot 2^3 = 2^{5+0+3} = 2^8$$

$$2^4 \cdot 2^5 \cdot 2^{\dots} = 2^9 \rightarrow 2^{9-4-5} = 2^0$$

De 2^{de} , 4^{de} (want $2^5 + 2^5 = 2 \cdot 2^5 = 2^{1+5} = 2^6$)
 en de 6^{de} .

11.2 HOEVEEL MOGELIJKHEDEN

2^3 routes ; 2^5 routes ; 2^8 routes
 $2^3 \cdot 2^5 = 2^8$

Bovenlangs is "aan" en "onderlangs" is uit.
 Van het eerste naar het tweede punt gaat over de bovenste lamp, van het tweede naar het derde punt over de lamp daaronder enzovoort ...

Alleen de bovenste lamp brandt.

$2^4 = 16$ signalen
 $2^7 = 128$ signalen

$2 \cdot 3 \cdot 2 = 12$ woorden
 122 ; 111 ; 213
 $3 \cdot 3 \cdot 3 = 27$ getallen

$3 \cdot 3 = 9$ torens



$3 \cdot 3 \cdot 3 = 27$ torens
 $3 \cdot 2 \cdot 2 \cdot 3 = 36$ torens

$2^{10} = 1024$ torens
 $10^2 = 100$ torens

11.3 REKENEN MET MACHTEN

Van $3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3$.

$$\begin{aligned} 3^2 \cdot 3^6 &= 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \\ &= 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \\ &= 3^8 \end{aligned}$$

$$3^5 \cdot 3^7 = 3^{5+7} = 3^{12}$$

$$3^4 \cdot 3^8 = 3^{4+8} = 3^{12}$$

$$3^5 \cdot 3^6 = 3^{5+6} = 3^{11}$$

$$3^4 \cdot 3 = 3^{4+1} = 3^5$$

$$3 \cdot 3^5 \cdot 3^7 = 3^{1+5+7} = 3^{13}$$

$$3^3 \cdot 3^4 \cdot 3^8 = 3^{3+4+8} = 3^{15}$$

$$3^4 \cdot 3^5 \cdot 3^6 = 3^{4+5+6} = 3^{15}$$

$$3^4 \cdot 3 \cdot 3^3 = 3^{4+1+3} = 3^8$$

Nee, $3^5 = 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 = 243$ en

$$5^3 = 5 \cdot 5 \cdot 5 = 125.$$

$2^7 = 128$, $7^2 = 49$, $4^5 = 1024$, $5^4 = 625$

Het kwadraat van 7.

$$6^4 = 1296$$

$$0^3 = 0 ; 1^3 = 1 ; 2^3 = 8 ; 3^3 = 27$$

$$4^3 = 64 ; 5^3 = 125 ; 6^3 = 216$$

$$0^4 = 0 ; 1^4 = 1 ; 2^4 = 16 ; 3^4 = 81$$

$$4^4 = 256$$

$$0^5 = 0 ; 1^5 = 1 ; 2^5 = 32 ; 3^5 = 243$$

$$64 = 2^6 ; 64 = 4^3 ; 64 = 8^2$$

$$1000 = 10^3 ; 8 = 2^3 ; 8000 = 20^3$$

$$\frac{2}{3} \cdot \frac{2}{3} \cdot \frac{2}{3} = \frac{8}{27} ; \frac{3}{4} \cdot \frac{3}{4} \cdot \frac{3}{4} = \frac{27}{64} ; \frac{2}{3} \cdot \frac{2}{3} \cdot \frac{2}{3} \cdot \frac{2}{3} = \frac{16}{81}$$

$$\frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10} = \frac{1}{10.000} ; \frac{11}{10} \cdot \frac{11}{10} \cdot \frac{11}{10} = \frac{1331}{1000} ;$$

$$\frac{1}{5} \cdot \frac{1}{5} \cdot \frac{1}{5} \cdot \frac{1}{5} = \frac{1}{625}$$

$$-3 \cdot -3 \cdot -3 \cdot -3 = 81 ; -2 \cdot -2 \cdot -2 = -8 ; 1$$

$$\begin{aligned} 2^2 \cdot 2^6 &= 2^{2+6} = 2^8 & 2^5 \cdot 4 &= 2^5 \cdot 2^2 = 2^{5+2} = 2^7 \\ 3^2 \cdot 3^6 &= 3^{2+6} = 3^8 & 3^2 \cdot 27 &= 3^2 \cdot 3^3 = 3^{2+3} = 3^5 \\ 5^2 \cdot 5^3 &= 5^{2+3} = 5^5 & 5^2 \cdot 25 &= 5^2 \cdot 5^2 = 5^{2+2} = 5^4 \end{aligned}$$

$$\begin{aligned} 10^3 \cdot 10^3 &= 10^{3+3} = 10^6 & 10^{11} \cdot 10.000 &= 10^{11} \cdot 10^4 = 10^{15} \\ 10^{n+3} & & 10^n \cdot 100 &= 10^n \cdot 10^2 = 10^{n+2} \\ 10^{n+m} & & 10^{n+1} & \end{aligned}$$

$$\left(\frac{1}{2}\right)^3 \cdot \left(\frac{1}{2}\right)^3 = \left(\frac{1}{2}\right)^{3+3} = \left(\frac{1}{2}\right)^6 \quad \left(\frac{3}{4}\right)^3 \cdot \left(\frac{3}{4}\right)^7 = \left(\frac{3}{4}\right)^{3+7} = \left(\frac{3}{4}\right)^{10}$$

$$(2^3)^3 = 2^3 \cdot 2^3 \cdot 2^3 = 2^{3 \cdot 3} = 2^9$$

$$(2^3)^4 = 2^3 \cdot 2^3 \cdot 2^3 \cdot 2^3 = 2^{4 \cdot 3} = 2^{12}$$

$$(2^4)^0 = 2^{4 \cdot 0} = 2^0$$

$$(3^2)^3 = 3^2 \cdot 3^2 \cdot 3^2 = 3^{3 \cdot 2} = 3^6$$

$$(3^3)^4 = 3^3 \cdot 3^3 \cdot 3^3 \cdot 3^3 = 3^{4 \cdot 3} = 3^{12}$$

$$(3^4)^0 = 3^{4 \cdot 0} = 3^0$$

$$(2^4)^{10} = 2^{4 \cdot 10} = 2^{40} \quad (2^5)^{10} = 2^{5 \cdot 10} = 2^{50}$$

$$(3^2)^{10} = 3^{2 \cdot 10} = 3^{20} \quad (3^3)^{15} = 3^{3 \cdot 15} = 3^{45}$$

$$(2^2)^{11} = 2^{2 \cdot 11} = 2^{22} \quad (2^3)^{15} = 2^{3 \cdot 15} = 2^{45}$$

$$a^{p \cdot 3} = a^{3p}$$

$$a^{p \cdot q} = a^{pq}$$

$3 \cdot 12 = 36$, dus $36 : 3 = 12$

$$3^6 ; 3^6$$

$$3^8 ; 3^8$$

$$3^{10} ; 3^{10}$$

$$3^{20} ; 3^{20}$$

$$3^{40} : 3^{10} = 3^{40-10} = 3^{30} \quad 3^{40} : 3^5 = 3^{40-5} = 3^{35}$$

$$3^{40} : 3^{40} = 3^{40-40} = 3^0 \quad 3^{40} : 3^{30} = 3^{40-30} = 3^{10}$$

$$\left(\frac{1}{2}\right)^{40-10} = \left(\frac{1}{2}\right)^{30} \quad \left(\frac{1}{2}\right)^{40-15} = \left(\frac{1}{2}\right)^{25}$$

$$\left(\frac{1}{2}\right)^{40-30} = \left(\frac{1}{2}\right)^{10} \quad \left(\frac{1}{2}\right)^{40-40} = \left(\frac{1}{2}\right)^0$$

$$\frac{2^4}{3^4} = \frac{16}{81} \quad \frac{3^5}{2^5} = \frac{243}{32}$$

$$\frac{2^6}{3^6} = \frac{64}{729} \quad \frac{3^7}{2^7} = \frac{2187}{128}$$

$$\frac{3^6}{10^6} = \frac{729}{1.000.000} \quad \frac{2^7}{10^7} = \frac{128}{10.000.000}$$

$$\frac{1^7}{3^7} = \frac{1}{2187} \quad \frac{10^6}{3^6} = \frac{1.000.000}{729}$$

100 ; 1000 ; 10000

$$2^3 \cdot 5^3 = 2 \cdot 2 \cdot 2 \cdot 5 \cdot 5 \cdot 5 = 2 \cdot 5 \cdot 2 \cdot 5 \cdot 2 \cdot 5 = 10 \cdot 10 \cdot 10 = 1000$$

$$0,2^3 \cdot 10^3 = 0,2 \cdot 0,2 \cdot 0,2 \cdot 10 \cdot 10 \cdot 10 = 0,2 \cdot 10 \cdot 0,2 \cdot 10 \cdot 0,2 \cdot 10 = 2 \cdot 2 \cdot 2 = 2^3$$

$$2^5 \cdot 3^5 = (2 \cdot 3)^5 = 6^5$$

$$\left(\frac{1}{9}\right)^5 \cdot 3^5 = \left(\frac{1}{9} \cdot 3\right)^5 = \left(\frac{1}{3}\right)^5$$

$$20^6 \cdot 0,1^6 = (20 \cdot 0,1)^6 = 2^6$$

$$20^7 \cdot \left(\frac{1}{4}\right)^7 = \left(20 \cdot \frac{1}{4}\right)^7 = 5^7$$

Linkerkolom:

$$\left(\frac{1}{9}\right)^3 \cdot 3^3 = \left(\frac{1}{9} \cdot 3\right)^3 = \left(\frac{1}{3}\right)^3 = \frac{1}{27}$$

$$\left(\frac{1}{9}\right)^3 \cdot 3^5 = \left(\frac{1}{9}\right)^3 \cdot 3^3 \cdot 3^2 = \frac{1}{27} \cdot 9 = \frac{1}{3}$$

$$\left(\frac{1}{2}\right)^{10} \cdot 2^{10} = \left(\frac{1}{2} \cdot 2\right)^{10} = 1^{10} = 1$$

Rechterkolom:

$$\left(\frac{1}{9}\right)^3 \cdot 3^4 = \left(\frac{1}{9}\right)^3 \cdot 3^3 \cdot 3 = \frac{1}{27} \cdot 3 = \frac{1}{9}$$

$$\left(\frac{1}{9}\right)^3 \cdot 3^6 = \left(\frac{1}{9}\right)^3 \cdot 3^3 \cdot 3^3 = \frac{1}{27} \cdot 27 = 1$$

$$\left(\frac{1}{4}\right)^4 \cdot \left(\frac{2}{3}\right)^4 = \left(\frac{1}{4} \cdot \frac{2}{3}\right)^4 = \left(\frac{1}{6}\right)^4 = \frac{1}{1296}$$

$$\left(\frac{1}{10}\right)^3 = \frac{1^3}{10^3} = \frac{1}{1000} \quad 1.000.000 \quad \frac{4^3}{5^3} = \frac{64}{125}$$

$$81 \cdot 1.000.000 = 81.000.000 \quad 1 \quad 1$$

$$1 \cdot 0 = 0 \quad 6 \cdot 1 = 6 \quad \left(3 \cdot \frac{1}{2}\right)^{30} = \left(1\frac{1}{2}\right)^{30}$$

$$(2 \cdot 3)^3 = 6^3$$

$$2^{4+3} = 2^7$$

$$2^{4-3} = 2^1$$

$$2^{40-30} = 2^{10}$$

$$2^{40-40} = 2^0$$

$$(2 \cdot 3)^{10} = 6^{10}$$

$$10^4$$

$$10^5$$

$$10^6$$

$$10^7$$

$$10^4$$

$$10^5$$

$$2^{27} = 67.108.864 \cdot 2 = 134.217.728$$

$$2^{25} = 67.108.864 : 2 = 33.554.432$$

SUPER OPGAVEN

$$8 \cdot 2 \cdot 2 = 32 \text{ uiteinden}$$

$$1024 \cdot 2 = 2048 \text{ uiteinden}$$

$2^{10} \cdot 10^3$ is 1024 · 1000 en dat is 1.024.000, dat schrijf je met 7 cijfers

2^{20} is 1024 · 1024, dat is iets meer 1 miljoen, dus 7 cijfers

$2^{10} + 2^{11}$ is 1024 + 2048 en dat is 3072, dus 4 cijfers

$2^{10} + 10^5$ is 1024 + 100.000 en dat is 101.024, dat schrijf je met 6 cijfers

$2^{30} + 10^5$ is 1024 · 1024 · 1024 + 100.000, dat is ongeveer 1 miljard + 100.000, dus 10 cijfers

Dat de uit spelende club wint.

De derde wedstrijd.

$$3^{13} = 1.594.323 \text{ manieren}$$

Omdat er zoveel mogelijkheden om in te vullen zijn.

$$2^{13} = 8192 \text{ manieren}$$

$$64 = 2^6 \quad 64 = 4^3 \quad 64 = 8^2$$

$$(2^6)^3 = 2^{6 \cdot 3} = 2^{18}$$

$$(4^3)^3 = 4^{3 \cdot 3} = 4^9$$

$$(8^2)^3 = 8^{2 \cdot 3} = 8^6$$

$$f^8 = (f^4)^2 = 25^2 = 625$$

$$f^2 = \sqrt{f^4} = \sqrt{25} = 5$$

$$f^6 = (f^2)^3 = 5^3 = 125$$

Linkerkolom:

$$(2 \cdot 5)^5 = 10^5 = 100.000$$

$$\left(\frac{1}{9} \cdot 9\right)^{30} = 1^{30} = 1$$

$$\left(\frac{1}{2} \cdot 2\right)^{10} = 1^{10} = 1$$

$$\left(\frac{1}{2}\right)^5 \cdot 2^5 \cdot 2^5 = 1^5 \cdot 2^5 = 1 \cdot 2^5 = 32$$

Rechterkolom:

$$2^5 \cdot 5^5 \cdot 5 = 10^5 \cdot 5 = 500.000$$

$$\left(\frac{1}{9}\right)^{30} \cdot 9^{30} \cdot 9 = 1^{30} = 1^{30} \cdot 9 = 9$$

$$\left(\frac{1}{2}\right)^{10} \cdot 20^{10} \cdot 10^{10} = 1 \cdot 10^{10} = 10.000.000.000$$

$$\left(\frac{1}{2}\right)^5 \cdot \left(\frac{1}{2}\right)^5 \cdot 2^5 = \left(\frac{1}{2}\right)^5 \cdot 1^5 = \left(\frac{1}{2}\right)^5 = \frac{1^5}{2^5} = \frac{1}{32}$$

4096 is het kwadraat van $2^6 = 64$

4096 is de derde macht van $2^4 = 16$

4096 is de vierde macht van $2^3 = 8$

4096 is geen vijfde macht (van een geheel getal)

$$4096 \cdot 4 = 2^{12} \cdot 2^2 = 2^{12+2} = 2^{14} = (2^2)^7 = 4^7$$

$$2^{12} \cdot \left(\frac{1}{2}\right)^6 = 2^6 \cdot 2^6 \cdot \left(\frac{1}{2}\right)^6 = 2^6 \cdot 1^6 = 2^6 = (2^2)^3 = 4^3$$

$$4096 \cdot \frac{1}{4} = 2^{12} : 2^2 = 2^{12-2} = 2^{10} = (2^2)^5 = 4^5$$

$$2^{12} \cdot \left(\frac{1}{4}\right)^6 = 2^{12} : (2^2)^6 = 2^{12} : 2^{12} = 2^0 = 4^0$$

11.5 EXTRA OPGAVEN

$$10^3 = 1000 \text{ loten}$$

$$26 \cdot 10 \cdot 10 = 2600 \text{ loten}$$

8 cijfers, met 7 cijfers kun je $10^7 = 10$ miljoen verschillende nummers maken en dat is te weinig, met 8 cijfers kun je $10^8 = 100$ miljoen verschillende nummers maken.

$$5 \cdot 3 \cdot 4 = 60 \text{ manieren}$$

$$4^6 = 4096 \text{ manieren}$$

$$4^4 = 256 \text{ manieren}$$

Na 1 minuut kennen 5 mensen het praatje. In de 2de minuut komen daar 20 mensen bij, dus kennen dan in totaal 25 mensen in praatje. In de 3de minuut komen daar 100 mensen bij, dus in totaal 125. In de 4de minuut komen daar 500 mensen bij, dus in totaal 625 mensen.

Je kunt het antwoord ook (sneller) berekenen, door steeds met 5 te vermenigvuldigen.

n	6	7	8
5^n	15.625	78.125	390.625

Na 8 minuten.

$$6^3 = 216 \text{ menu's}$$

$$5^3 = 125 \text{ menu's}$$

$$4^4 = 256 \text{ signalen}$$

$$3^4 = 81 \text{ signalen}$$

$$5^3 = 125 \text{ woorden}$$

$$(10^3)^2 = 10^{3 \cdot 2} = 10^6 \quad 10^2 \cdot 10^5 = 10^{2+5} = 10^7$$

$$10 \cdot 10^6 = 10^{1+6} = 10^7 \quad 10 \cdot 10^9 = 10^{10}$$

$$(10^6)^2 = 10^{6 \cdot 2} = 10^{12} \quad (10^{10})^2 = 10^{10 \cdot 2} = 10^{20}$$

$$10^6$$

$$10^7$$

$$10^9$$

$$10^{10}$$

$$10^6$$

$$10^7$$

Linkerkolom:

$$16^3 = (2^4)^3 = 2^{4 \cdot 3} = 2^{12}$$

$$(2^3)^a = 2^{3 \cdot a} = 2^{3a}$$

$$2^a \cdot 64^3 = 2^a \cdot (2^6)^3 = 2^a \cdot 2^{18} = 2^{a+18}$$

$$2^a \cdot 2 = 2^a \cdot 2^1 = 2^{a+1}$$

Rechterkolom:

$$64^3 = (2^6)^3 = 2^{6 \cdot 3} = 2^{18}$$

$$2^a \cdot 2^3 = 2^{a+3}$$

$$2^a \cdot 2^a = 2^{a+a} = 2^{2a}$$

$$2^a : 2^a = 2^{a-a} = 2^0$$

$$2^{10} : 2^5 = 2^{10-5} = 2^5 \quad 2^8 : 2^5 = 2^{8-5} = 2^3$$

$$2^6 : 2^5 = 2^{6-5} = 2^1 \quad 2^5 : 2^5 = 2^{5-5} = 2^0$$

$$2^{10:2} = 2^5 \quad 2^{5:5} = 2^1$$

$$3$$

$$9$$

$$2$$

$$4$$

$$8$$

$$\left(\frac{1}{3}\right)^3 = \frac{1^3}{3^3} = \frac{1}{27}$$

$$\left(\frac{10}{3}\right)^3 = \frac{10^3}{3^3} = \frac{1000}{27}$$

$$\left(\frac{100}{3}\right)^3 = \frac{100^3}{3^3} = \frac{1.000.000}{27}$$