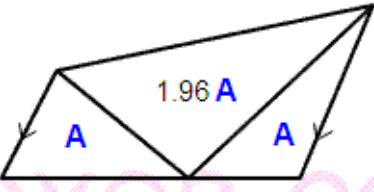


Solution

349 ©www.8foxes.com

A trapezoid (AmE) / معين منحرف (Ara) / Trapezium (BrE) / Rozloha (Che) / τραπέζιο (Gr) / טרפז (Heb) / 台形 (Jap) / Yamuk (Tur) is divided into 3 pieces as below:



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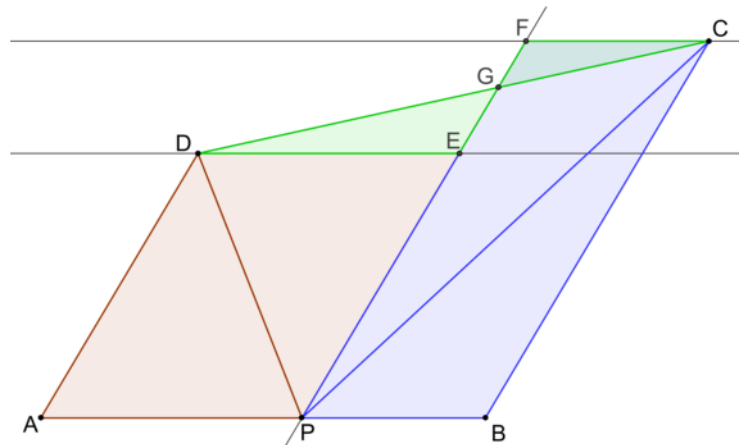
PROVE that no, this won't happen in any language.

Name the trapezoid $ABCD$ with $AD \parallel BC$.

P the point on AB with $\text{Area}(APD) = \text{Area}(BPC) = A$.

We take $d(D, AB) < d(C, AB)$.
See picture.

Draw parallel lines through D, P, C , giving E, F and G .



$$\text{Area}(APD) = \text{Area}(BPC)$$

$$\text{so } \frac{1}{2} \cdot AP \cdot d(D, AB) = \frac{1}{2} \cdot BP \cdot d(C, AB) \rightarrow AP > BP, \text{ because } d(D, AB) < d(C, AB).$$

$\triangle APD = \triangle EDP$ and $\triangle BPC = \triangle FCP$ (a parallelogram is divided in two equal areas by its diagonal).

And $\triangle GED \sim \triangle GFC$ (because of the parallel lines giving equal angles)

Because $DE = AP > BP = FC$, is $\text{Area}(DEG) > \text{Area}(CFG)$.

So $\text{Area}(DEG) - \text{Area}(CFG) > 0$.

$$\begin{aligned} \text{Area}(CDP) &= \text{Area}(DEP) + \text{Area}(PCG) + \text{Area}(DEG) \\ &= \text{Area}(DEP) + \text{Area}(FCP) - \text{Area}(CFG) + \text{Area}(DEG) \\ &= \text{Area}(APD) + \text{Area}(BPC) + (\text{Area}(DEG) - \text{Area}(CFG)) \\ &> A + A \\ &= 2A \\ &> 1.96A \end{aligned}$$

QED.