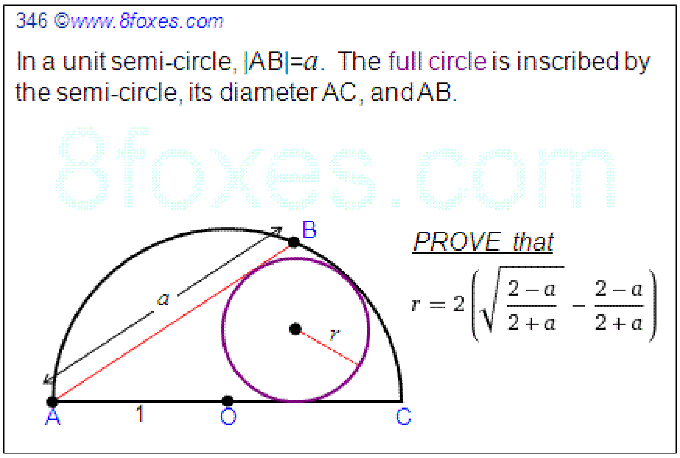
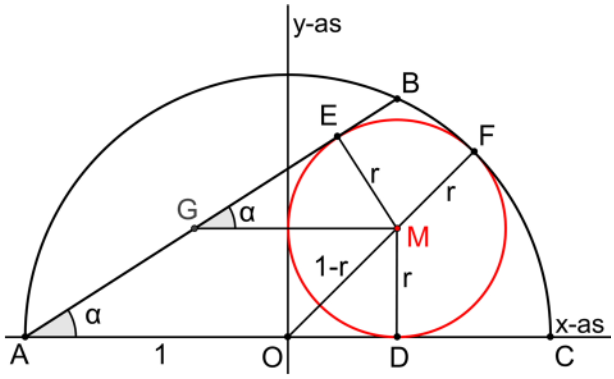


Solution Fox346



See figure for names.
$\angle MEA = \angle MDA = 90^\circ$ (tangents to circle).
$\angle ABC = 90^\circ$ (Thales),

so in triangle ABC: $\cos\alpha = \dfrac{AB}{AC} = \dfrac{a}{2}$.

$$\sin\alpha = \sqrt{1-\cos^2\alpha} = \sqrt{1-\left(\tfrac{a}{2}\right)^2}$$
$$= \sqrt{1-\tfrac{1}{4}a^2} = \tfrac{1}{2}\sqrt{4-a^2}$$

Pythagoras in ΔODM: $OD = \sqrt{(1-r)^2 - r^2} = \sqrt{1-2r}$.

Coordinates of $O(0, 0)$, $A(-1, 0)$, $C(1, 0)$, $M(\sqrt{1-2r}, r)$.

Parallel line through M gives point G on AB, then $\angle MGE = \angle CAB = \alpha$ (F-angles) and $\angle GME = 90^\circ - \alpha$.

$$x_E = x_M - r\cdot\cos(90^\circ-\alpha) = \sqrt{1-2r} - r\cdot\sin\alpha$$
$$y_E = y_M + r\cdot\sin(90^\circ-\alpha) = r + r\cdot\cos\alpha\text{, so } E = (\sqrt{1-2r} - r\sin\alpha,\ r + r\cos\alpha).$$

Formula of line AB: $y = \tan(\alpha)\cdot(x+1)$, or $y\cdot\cos\alpha = \sin\alpha\cdot(x+1)$.

E lies on AB, so: $(r + r\cos\alpha)\cdot\cos\alpha = \sin\alpha\cdot(\sqrt{1-2r} - r\sin\alpha + 1)$

$$r\cos\alpha + r\cos^2\alpha + r\sin^2\alpha = r\cos\alpha + r(\cos^2\alpha + \sin^2\alpha) = \sin\alpha\cdot\sqrt{1-2r} + \sin\alpha$$
$$r\cos\alpha + r - \sin\alpha = \sin\alpha\cdot\sqrt{1-2r}$$

Now substituting $\cos\alpha = \frac{a}{2}$ and $\sin\alpha = \frac{1}{2}\sqrt{4-a^2}$ and multiplying by 2:

$$ra + 2r - \sqrt{4-a^2} = \sqrt{4-a^2}\cdot\sqrt{1-2r}$$
$$r(2+a) - \sqrt{(2-a)(2+a)} = \sqrt{(2-a)(2+a)}\cdot\sqrt{1-2r}$$

Dividing by $\sqrt{(2-a)(2+a)}$ gives $r\sqrt{\dfrac{2+a}{2-a}} - 1 = \sqrt{1-2r}$

Quadrating: $r^2\cdot\dfrac{2+a}{2-a} - 2r\sqrt{\dfrac{2+a}{2-a}} + 1 = 1 - 2r$

Rearranging and dividing by r gives: $r\cdot\dfrac{2+a}{2-a} = 2\left(\sqrt{\dfrac{2+a}{2-a}} - 1\right)$

$$r = 2\left(\sqrt{\frac{2+a}{2-a}} - 1\right)\cdot\frac{2-a}{2+a} = 2\left(\frac{2-a}{2+a}\cdot\sqrt{\frac{2+a}{2-a}} - \frac{2-a}{2+a}\right) = 2\left(\sqrt{\frac{2-a}{2+a}} - \frac{2-a}{2+a}\right)$$

QED.