

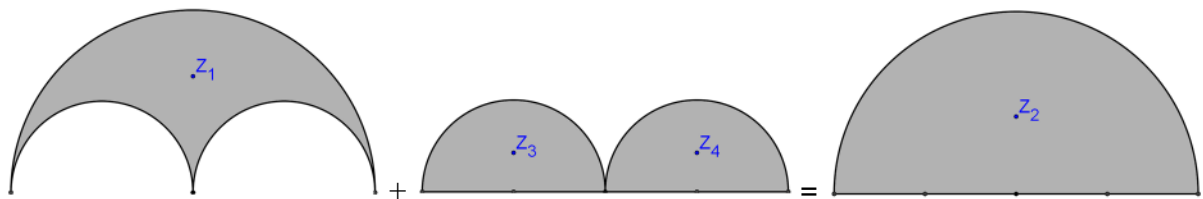
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Unit circle is converted into a yin-yang. Assume that the white piece (yang) was cut out of the big circle. Ignoring the dots, what is the coordinates of yin's centroid (blue dot)?

A)  $(1/6, 1/\pi)$   
 B)  $(1/5, 2/\pi)$   
 C)  $(1/5, 1/\pi)$   
 D)  $(1/4, 2/\pi)$   
 E)  $(1/4, 1/\pi)$

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**Solution:**



$$m_1 = \frac{1}{2}(\pi \cdot 1^2 - 2 \cdot \pi \cdot (\frac{1}{2})^2) = \frac{\pi}{4}$$

$$m_3 = m_4 = \frac{1}{2} \cdot \pi \cdot (\frac{1}{2})^2 = \frac{\pi}{8}$$

$$m_2 = \frac{1}{2} \cdot \pi \cdot 1^2 = \frac{\pi}{2}$$

$$y_1 = ?$$

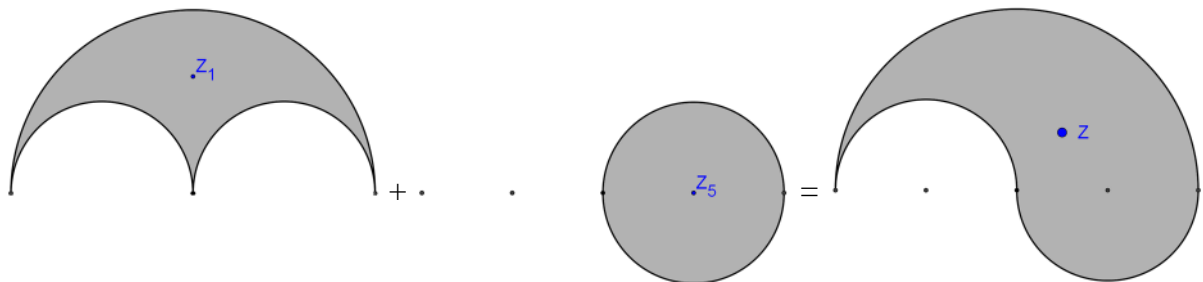
$$y_3 = y_4 = \frac{2}{3\pi}$$

$$y_2 = \frac{4}{3\pi}$$

$$\text{So: } y_1 \cdot m_1 + y_3 \cdot m_3 + y_4 \cdot m_4 = y_2 \cdot m_2 \rightarrow y_1 \cdot \frac{\pi}{4} + 2 \cdot \frac{2}{3\pi} \cdot \frac{\pi}{8} = \frac{4}{3\pi} \cdot \frac{\pi}{2} \rightarrow y_1 \cdot \frac{\pi}{4} = \frac{2}{3} - \frac{1}{6} = \frac{1}{2}$$

$$\text{And we find } y_1 = \frac{2}{\pi}$$

Further  $x_1 = 0$ , because of the symmetry.



$$m_1 = \frac{\pi}{4}$$

$$m_5 = \pi \cdot (\frac{1}{2})^2 = \frac{\pi}{4}$$

$$m_z = \frac{\pi}{4} + \frac{\pi}{4} = \frac{\pi}{2}$$

$$y_1 = \frac{2}{\pi}$$

$$y_5 = 0$$

$$y_z = ?$$

$$x_1 = 0$$

$$x_5 = \frac{1}{2}$$

$$x_z = ?$$

Because  $m_1 = m_5$ , is the centroid Z the midpoint of  $Z_1$  and  $Z_5$ .

$$x_z = \frac{1}{2}(x_1 + x_5) = \frac{1}{4} \quad \text{and} \quad y_z = \frac{1}{2}(y_1 + y_5) = \frac{1}{\pi}$$

The correct answer is E:  $Z = \left(\frac{1}{4}, \frac{1}{\pi}\right)$