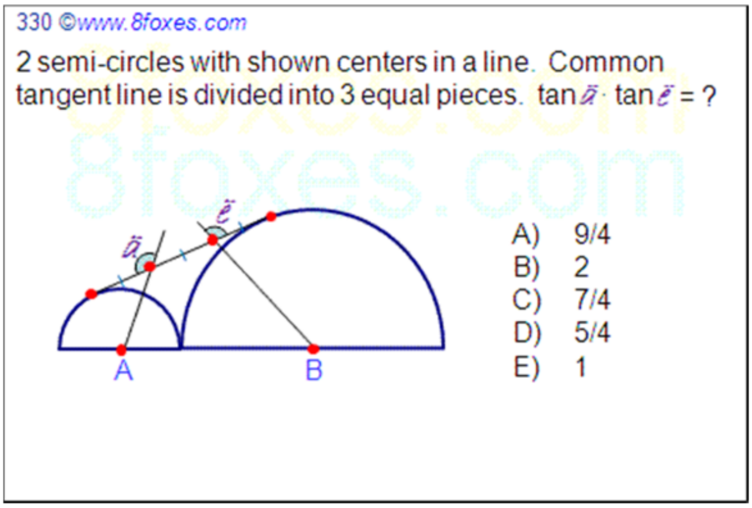
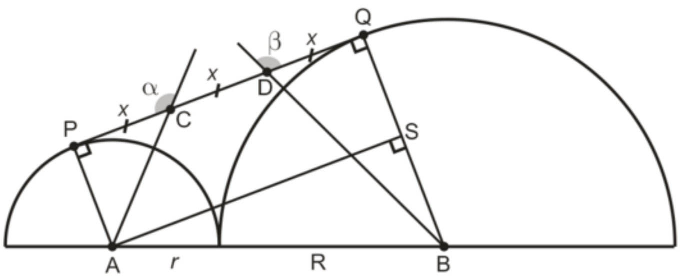
SOLUTION



See figure for naming of the points and edges.



$\angle ACP = 180^o - \alpha$ and $\angle BDQ = 180^o - \beta$.
Because PQ is tangent to both circles, PQ is perpendicular to both radiuses AP and BQ.
Therefore $\angle PAC = \alpha - 90^o$ and $\angle QBD = \beta - 90^o$.

We know the identities: $\tan(\varphi - 90°) = -\tan(90° - \varphi) = -\dfrac{\sin(90° - \varphi)}{\cos(90° - \varphi)} = -\dfrac{\cos(\varphi)}{\sin(\varphi)} = -\dfrac{1}{\tan\varphi}$.

Therefore: $\tan(\angle PAC) = \tan(\alpha - 90°) = -\dfrac{1}{\tan\alpha} = \dfrac{x}{r} \rightarrow \tan\alpha = -\dfrac{r}{x}$

and $\tan(\angle QBD) = \tan(\beta - 90°) = -\dfrac{1}{\tan\beta} = \dfrac{x}{R} \rightarrow \tan\beta = -\dfrac{R}{x}$.

From point A to radius BQ gives point S.
Pythagoras in triangle ABS: $AS^2 = AB^2 - BS^2 \rightarrow (3x)^2 = (R+r)^2 - (R-r)^2 = \ldots = 4rR$.
This gives $9x^2 = 4rR \rightarrow rR = {}^9/_4 \cdot x^2$.

$$\tan\alpha \cdot \tan\beta = -\frac{r}{x} \cdot -\frac{R}{x} = \frac{rR}{x^2} = \frac{\frac{9}{4}x^2}{x^2} = {}^9\!/_4.$$

The answer is: A