

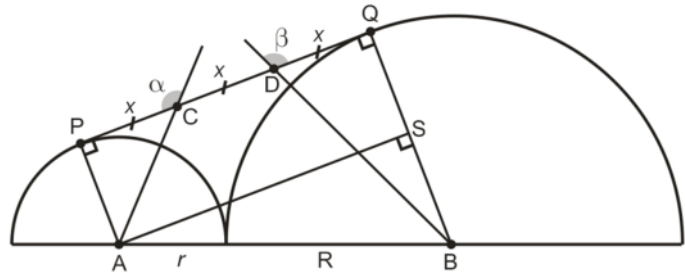
SOLUTION

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 2 semi-circles with shown centers in a line. Common tangent line is divided into 3 equal pieces.  $\tan \tilde{\alpha} \cdot \tan \tilde{\beta} = ?$

A) 9/4  
 B) 2  
 C) 7/4  
 D) 5/4  
 E) 1

See figure for naming of the points and edges.

$\angle ACP = 180^\circ - \alpha$  and  $\angle BDQ = 180^\circ - \beta$ .  
 Because  $PQ$  is tangent to both circles,  $PQ$  is perpendicular to both radii  $AP$  and  $BQ$ .



Therefore  $\angle PAC = \alpha - 90^\circ$  and  $\angle QBD = \beta - 90^\circ$ .

We know the identities:  $\tan(\varphi - 90^\circ) = -\tan(90^\circ - \varphi) = -\frac{\sin(90^\circ - \varphi)}{\cos(90^\circ - \varphi)} = -\frac{\cos(\varphi)}{\sin(\varphi)} = -\frac{1}{\tan \varphi}$ .

Therefore:  $\tan(\angle PAC) = \tan(\alpha - 90^\circ) = -\frac{1}{\tan \alpha} = \frac{x}{r} \rightarrow \tan \alpha = -\frac{r}{x}$

and  $\tan(\angle QBD) = \tan(\beta - 90^\circ) = -\frac{1}{\tan \beta} = \frac{x}{R} \rightarrow \tan \beta = -\frac{R}{x}$ .

From point  $A$  to radius  $BQ$  gives point  $S$ .

Pythagoras in triangle  $ABS$ :  $AS^2 = AB^2 - BS^2 \rightarrow (3x)^2 = (R+r)^2 - (R-r)^2 = \dots = 4rR$ .

This gives  $9x^2 = 4rR \rightarrow rR = \frac{9}{4}x^2$ .

$$\tan \alpha \cdot \tan \beta = -\frac{r}{x} \cdot -\frac{R}{x} = \frac{rR}{x^2} = \frac{\frac{9}{4}x^2}{x^2} = \frac{9}{4}$$

The answer is: A