

SOLUTION

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 A random process starts from point O . Point A is obtained in a **uniformly** selected direction, 1 unit apart from O . The same process is repeated for point A to obtain point B , and for point B to obtain point C . What is the probability that a triangle is obtained when point C is generated?

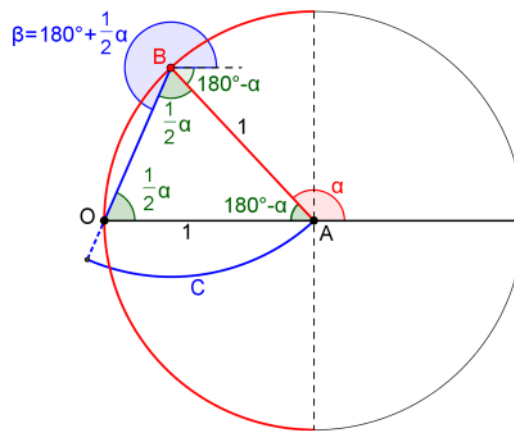
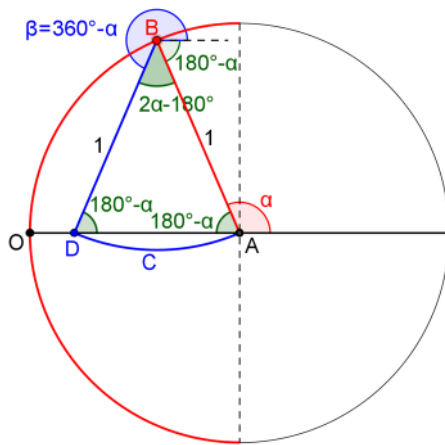
A) 1/8
 B) 1/9
 C) 1/12
 D) 1/16
 E) 1/18

We name α the angle moving from A to B and β the angle moving from B to C with $0^\circ \leq \alpha, \beta \leq 360^\circ$, measuring the conventional way (pointing right = angle 0° , pointing left = angle 180°).

It is clear that B is lying on the circle with radius 1 from A .

It is clear that that B must lie on the left half of this circle (see figure, the red half circle) to make it possible to make a triangle, so it must be $90^\circ < \alpha < 270^\circ$.

- For α , with $90^\circ < \alpha < 120^\circ$ the circle with radius 1 and center B intersects OA , in A and another point between O and A . Call this point D . Triangle ABD is isosceles with top-angle $2\alpha - 180^\circ$. For angles β with $360^\circ - \alpha < \beta < (360^\circ - \alpha) + (2\alpha - 180^\circ) = 180^\circ + \alpha$ point C creates a triangle.
- For α , with $120^\circ < \alpha < 180^\circ$ triangle OAB is isosceles with angle $OBC = \frac{1}{2}\alpha$. For angles β with $180^\circ + \frac{1}{2}\alpha < \beta < (180^\circ + \frac{1}{2}\alpha) + \frac{1}{2}\alpha = 180^\circ + \alpha$ point C creates a triangle.
- For $180^\circ < \alpha < 270^\circ$ the situations are (almost) the same because of the symmetry.

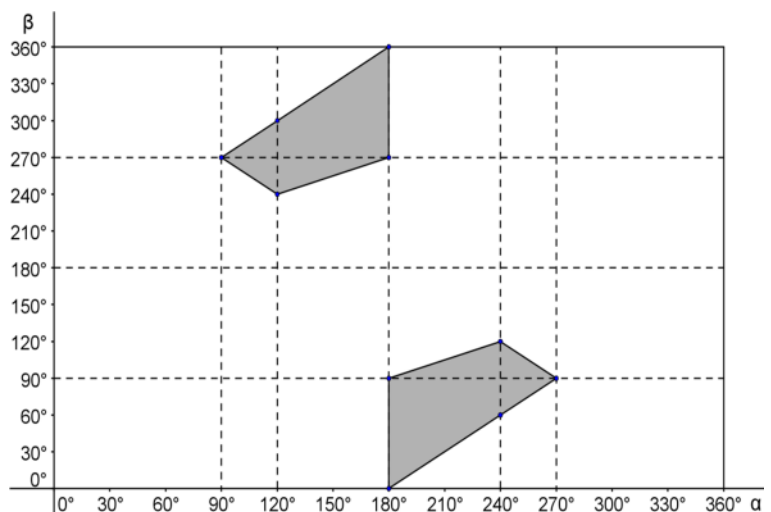


In the diagram we see a rectangle $[0^\circ, 360^\circ] \times [0^\circ, 360^\circ]$ with all possible combinations of α and β , corresponding with probability 1.

In the gray colored areas we find the combinations of α and β with C creating a triangle.

With easy calculation you can find that this gray area is $\frac{1}{12}$ of the whole area.

So the probability that a triangle is obtained when point C is generated is $\frac{1}{12}$.



The answer is C.