Fox 132

SOLUTION



We watch the biggest square in the middle, see figure:

- Triangles *ADE*, *ECF* and *ABC* are similar, so $EC = 2 \cdot \frac{x}{\sqrt{5}} = \frac{2}{5}\sqrt{5} \cdot x$ and $AE = x \cdot \sqrt{5}$
- AE + EC = AC = 2, so $\frac{2}{5}\sqrt{5} \cdot x + \sqrt{5} \cdot x = \frac{7}{5}\sqrt{5} \cdot x = 2$ $\rightarrow x = \frac{2}{7}\sqrt{5}$ and area $DEFG = x^2 = \frac{20}{49}$.
- $EC = \frac{4}{7}$; $AE = 2 \frac{4}{7} = \frac{10}{7}$

•
$$CF = \frac{2}{7}; BF = 1 - \frac{2}{7} = \frac{5}{7}$$

Now we watch any right-angled triangle similar to *ABC*, with a square in the right angle.

When the square has sides *x*, then it's easy to see that the two remaining triangles are similar with the original triangle, one with factor $^{2}/_{3}$ and the other with factor $^{1}/_{3}$. See figure.

And it follows that de side $x = \frac{\text{hypothenusa}}{1\frac{1}{2}\sqrt{5}} = \frac{2}{3\sqrt{5}} \cdot \text{hypothenusa}$.

Now we distinguish between the left and right side of the figure:

Right side:

In every step the triangle gets smaller with factor $\frac{1}{3}$, so also every square gets smaller with factor $\frac{1}{3}$. The first square has side $x_0 = \frac{2}{3\sqrt{5}} \cdot BF = \frac{2}{3\sqrt{5}} \cdot \frac{5}{7} = \frac{2\sqrt{5}}{21}$ and area $(x_0)^2 = \frac{20}{441}$.

The sides of the squares on the right side give the following geometric sequence:

$$\frac{2\sqrt{5}}{21}, \ \frac{2\sqrt{5}}{21} \cdot \frac{1}{3}, \ \frac{2\sqrt{5}}{21} \cdot \left(\frac{1}{3}\right)^2, \ \frac{2\sqrt{5}}{21} \cdot \left(\frac{1}{3}\right)^3, \ \dots, \text{ or } x_n = \frac{2\sqrt{5}}{21} \cdot \left(\frac{1}{3}\right)^n \ (n = 0, 1, 2, \dots)$$

The area of these squares are $(x_n)^2 = \frac{20}{441} \cdot \left(\frac{1}{3}\right)^{2n} = \frac{20}{441} \cdot \left(\frac{1}{9}\right)^n$ (n = 0, 1, 2, ...). The total area of the squares on the right side is equal to

$$\sum_{n=0}^{\infty} \left(x_n\right)^2 = \sum_{n=0}^{\infty} \frac{20}{441} \cdot \left(\frac{1}{9}\right)^n = \frac{20}{441} \cdot \sum_{n=0}^{\infty} \left(\frac{1}{9}\right)^n = \frac{20}{441} \cdot \frac{1}{1 - \frac{1}{9}} = \frac{20}{441} \cdot \frac{9}{8} = \frac{5}{98}$$

Left side:

In every step the triangle, and also the square, is multiplied with factor $^2/_3$. The first square has side $y_0 = \frac{2}{3\sqrt{5}} \cdot AE = \frac{2}{3\sqrt{5}} \cdot \frac{10}{7} = \frac{4\sqrt{5}}{21}$ and area $(y_0)^2 = \frac{80}{441}$.

The sides of the squares on the left side give the following geometric sequence:

$$\frac{4\sqrt{5}}{21}, \ \frac{4\sqrt{5}}{21} \cdot \frac{2}{3}, \ \frac{4\sqrt{5}}{21} \cdot \left(\frac{2}{3}\right)^2, \ \frac{4\sqrt{5}}{21} \cdot \left(\frac{2}{3}\right)^3, \ \dots, \text{ or } \ y_n = \frac{4\sqrt{5}}{21} \cdot \left(\frac{2}{3}\right)^n \ (n = 0, 1, 2, \dots)$$

The total area of the squares on the left side is equal to

$$\sum_{n=0}^{\infty} \left(y_n\right)^2 = \sum_{n=0}^{\infty} \frac{80}{441} \cdot \left(\frac{4}{9}\right)^n = \frac{80}{441} \cdot \sum_{n=0}^{\infty} \left(\frac{4}{9}\right)^n = \frac{80}{441} \cdot \frac{1}{1 - \frac{4}{9}} = \frac{80}{441} \cdot \frac{9}{5} = \frac{16}{49}$$

- The total sum of the areas of all squares = $\frac{20}{49} + \frac{16}{49} + \frac{5}{98} = \frac{40}{98} + \frac{32}{98} + \frac{5}{98} = \frac{77}{98} = \frac{11}{14}$
- The answer is A

