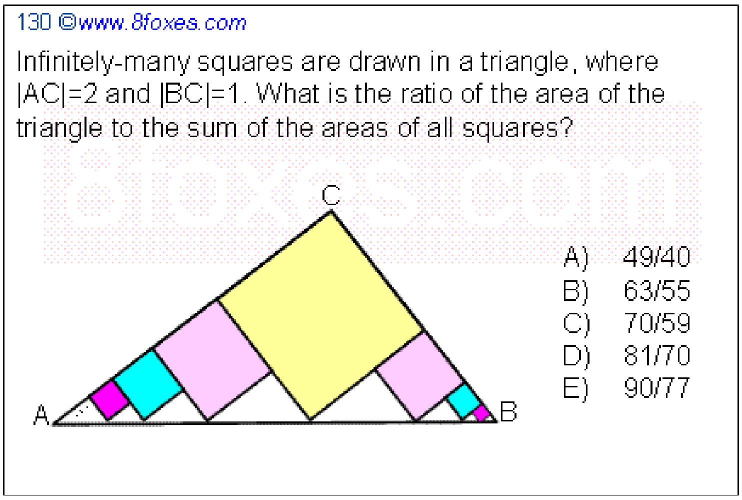
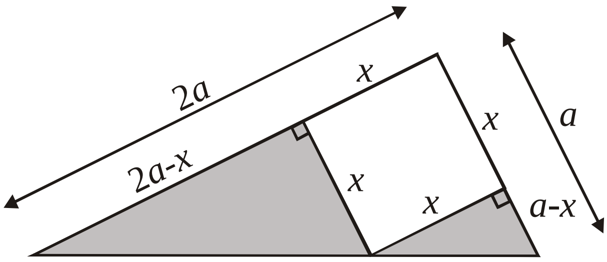
SOLUTION





When a square with sides x is cut off from a right-angled triangle with sides a and $2a$ (also similar to triangle ABC), then the remaining triangles are also similar to triangle ABC (same angles).

- For the <u>right</u> grey triangle:
  $\frac{x}{a-x}=\frac{2}{1} \;\rightarrow\; 2(a-x)=x \;\rightarrow\; x={}^2/_3a.$ And for the triangle the side: $a-x=a-{}^2/_3a={}^1/_3a.$
  So the <u>right</u> grey triangle is a multiple of the original triangle with factor $^1/_3$.
- For the side of the <u>left</u> grey triangle: $2a-x=2a-{}^2/_3a={}^4/_3a={}^2/_3\cdot 2a.$
  So the <u>left</u> grey triangle is a multiple of the original triangle with factor $^2/_3$.

We distinguish between the left and right side:

- Right side:
  For the first square the above gives $x={}^2/_3$. In every step the triangle gets smaller with factor $^1/_3$, so also every square gets smaller with factor $^1/_3$.
  The sides of the squares on the right side give the following geometric sequence:
  $\frac{2}{3},\ \frac{2}{3}\cdot\frac{1}{3},\ \frac{2}{3}\cdot\left(\frac{1}{3}\right)^2,\ \frac{2}{3}\cdot\left(\frac{1}{3}\right)^3,\ \ldots$, or $x_n=\frac{2}{3}\cdot\left(\frac{1}{3}\right)^n\ (n=0,1,2,\ldots)$
  The area of these squares are $(x_n)^2=\frac{4}{9}\cdot\left(\frac{1}{3}\right)^{2n}=\frac{4}{9}\cdot\left(\frac{1}{9}\right)^n\ (n=0,1,2,\ldots).$
  The *total area of the squares on the right side* is equal to
  $$\sum_{n=0}^{\infty}(x_n)^2=\sum_{n=0}^{\infty}\frac{4}{9}\cdot\left(\frac{1}{9}\right)^n=\frac{4}{9}\cdot\sum_{n=0}^{\infty}\left(\frac{1}{9}\right)^n=\frac{4}{9}\cdot\frac{1}{1-\frac{1}{9}}=\frac{4}{9}\cdot\frac{9}{8}=\frac{1}{2}$$
- Left side:
  In every step the triangle, and also the square, is multiplied with factor $^2/_3$.
  The sides of the squares on the left side give the following geometric sequence:
  $\frac{2}{3},\ \frac{2}{3}\cdot\frac{2}{3},\ \frac{2}{3}\cdot\left(\frac{2}{3}\right)^2,\ \frac{2}{3}\cdot\left(\frac{2}{3}\right)^3,\ \ldots$, or $y_n=\frac{2}{3}\cdot\left(\frac{2}{3}\right)^n\ (n=0,1,2,\ldots)$
  The *total area of the squares on the left side* is equal to
  $$\sum_{n=0}^{\infty}(y_n)^2=\sum_{n=0}^{\infty}\frac{4}{9}\cdot\left(\frac{4}{9}\right)^n=\frac{4}{9}\cdot\sum_{n=0}^{\infty}\left(\frac{4}{9}\right)^n=\frac{4}{9}\cdot\frac{1}{1-\frac{4}{9}}=\frac{4}{9}\cdot\frac{9}{5}=\frac{4}{5}$$
- But the first square with sides $^2/_3$ (and area $^4/_9$) is counted on both sides!
- The total sum of the areas of all squares = $\frac{1}{2}+\frac{4}{5}-\frac{4}{9}=\frac{77}{90}$
- The area of triangle ABC is equal to 1, so the ratio of the area of the triangle to the sum of all squares is $1 : {}^{77}/_{90}={}^{90}/_{77}$
- The answer is **E**